

**Estudio Analítico - Gráfico  
de los  
Poliedros Regulares  
y de sus  
Derivados y Conjugados**

**LAMINAS 26 AL 32**

**V**

**V**

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514  
ALV-V  
600107502

**Prof. T. Alvarez Peralfo**









A) Por proyección de los centros de caras.

R. 7815

19025117

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro derivado de un exaedro regular, obtenido al proyectar desde el centro de la esfera circunscrita a éste, y sobre ella, los centros de cada cara, uniendo a continuación estos puntos con los vértices del polígono de dicha cara.

Las coordenadas del centro de la esfera son:  $O(72, 72, 85)$  mm y el radio de la misma es 55 mm.

Dibujar en formato A3v y a escala 1:1.

DATOS $O(72, 72, 85)$  mm $a_6 = 55$  mm

A  
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V



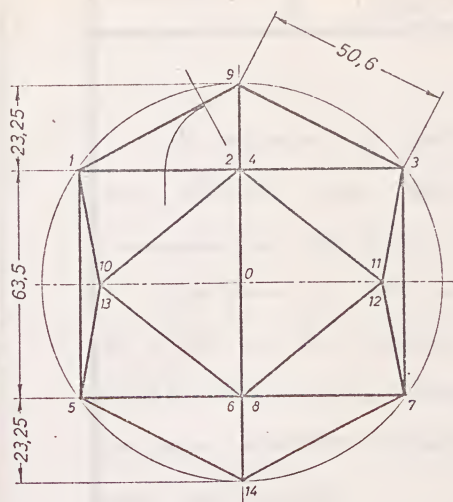
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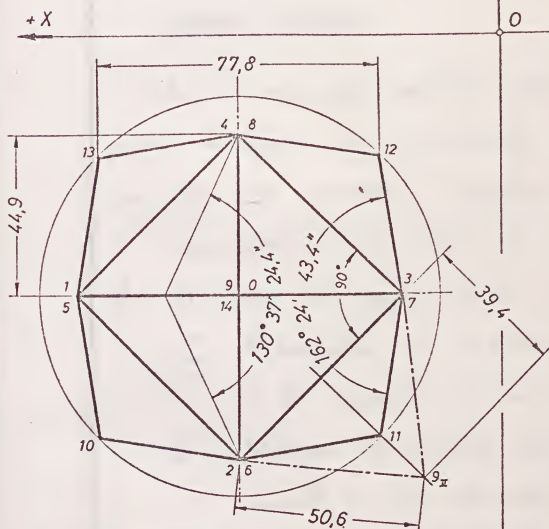
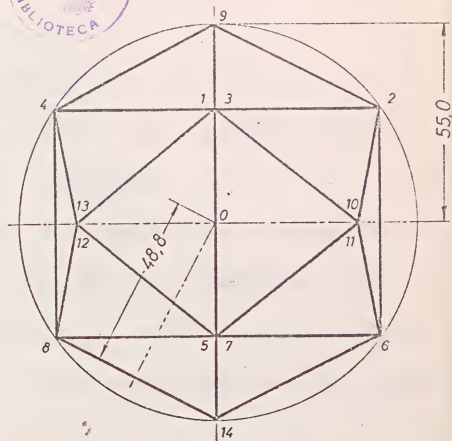
The following is a list of the names of the persons who have been elected to the office of the President of the United States since the year 1789. The names are given in alphabetical order, and the year of election is given in parentheses. The names are given in the order in which they were elected, and the year of election is given in parentheses.



I



III



#### NUMERACION DE VERTICES

Exaedro regular..... 1 al 8  
Proyecciones centros caras del mismo  
(vértices del octaedro conjugado)..... 9 al 14

#### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro derivado de un exaedro regular, obtenido al proyectar desde el centro de la esfera circunscrita a éste, y sobre ella, los centros de cada cara, uniendo a continuación estos puntos con los vértices del polígono de dicha cara.

Las coordenadas del centro de la esfera son: O (72, 72, 85) mm y el radio de la misma de 55 mm.

Dibujar en formato A3v y a escala 1:1.

II

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala	Poliedro derivado del exaedro regular				
1:1	Lámina 26				
	Curso 19 - 20				



The following is a list of the  
 names of the persons who  
 have been appointed to the  
 various offices of the  
 Board of Education for the  
 year 1888-89. The names  
 are given in alphabetical  
 order, and the offices to  
 which they are appointed are  
 given in parentheses.  
 The names of the persons  
 who have been appointed to  
 the offices of the Board of  
 Education for the year 1888-89  
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 for the year 1888-89 are given  
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Al estudiar el ejercicio propuesto en la lámina 25, hemos obtenido unas deducciones puras de carácter general, comunes a los cinco poliedros regulares.

Las fórmulas allí deducidas las aplicaremos sucesivamente a los cuatro poliedros que quedan por estudiar. El desarrollo del cálculo correspondiente a esta lámina, seguirá pues aquellas directrices, a las que haremos las oportunas referencias.

#### PROCESO GRÁFICO

En el caso del poliedro derivado del exaedro regular, el proceso gráfico es inmediato, ya que sabemos que el conjugado del exaedro regular es el octaedro regular, y esta representación ha sido ya efectuada en el ejercicio de la lámina 23, cuyo proceso nos permite:

- 1º Representar el exaedro regular dado, de vértices 1 al 8, inscrito en una esfera de 55 mm de radio.
- 2º Obtener los vértices del octaedro conjugado 9 al 14, inscrito en la misma esfera (estos vértices se han de corresponder con los 11 al 16 de la lámina 23).
- 3º Unir los vértices 9 al 14 con los correspondientes de cada cara del exaedro dado.

Al terminar la representación del poliedro derivado, po-



I have the honor to acknowledge the receipt of your letter of the 14th inst. in relation to the matter of the purchase of the land for the proposed new building for the University of Chicago. I am glad to hear that you are interested in the project and that you are willing to contribute to the fund for the purchase of the land. I am sure that the University will be very grateful for your contribution and that the land will be purchased as soon as possible.

Very truly yours,

John D. Rockefeller  
President of the University of Chicago

Enclosed for you are two copies of the report of the Committee on the Purchase of the Land for the proposed new building for the University of Chicago. I am sure that you will find the report very interesting and that it will give you a full and complete understanding of the matter.

Very truly yours,

John D. Rockefeller  
President of the University of Chicago

demostrar que éste es un poliedro convexo, de

$$C = 4 \times 6 = 24 \text{ caras; (ver lám. 25, fórmula [1]); de}$$

$$V = 8 + 6 = 14 \text{ vértices (ver lám. 25, fórmula [2]); y de}$$

$$A = 12 + 4 \times 6 = 36 \text{ aristas (ver lám. 25, fórmula [3]).}$$

La demostración de la convexidad de este poliedro la haremos analíticamente.

#### PROCESO GRÁFICO-ANALÍTICO

Calculamos previamente los siguientes valores deducidos de ejercicios anteriores, en función del radio  $a_6$  (dato) de la esfera circunscrita al exaedro regular dado.

Número de caras "n" del exaedro dado

$$n = 6$$

Radio " $a_6$ " de la esfera circunscrita al mismo (dato del ejercicio).

Lado " $l_6$ " del exaedro dado

Se deduce de la fórmula 11, lám. 2.

$$l_6 = \frac{2}{\sqrt{3}} a_6 = \frac{2\sqrt{3}}{3} a_6$$

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First main paragraph of handwritten text, starting with "The ... of ..."

Second main paragraph of handwritten text, continuing the narrative or argument.

Third main paragraph of handwritten text, concluding the section.

Radio " $c_6$ " de la esfera inscrita en el mismo

Se deduce de la fórmula 13, lám. 2.

$$c_6 = \frac{1}{2} l_6 = \frac{1}{2} \times \frac{2\sqrt{3}}{3} a_6 = \frac{\sqrt{3}}{3} a_6$$

Radio " $d_6$ " de la circunferencia circunscrita al polígono regular de una cara del mismo.

Se deduce de la fórmula 14, lám. 2

$$d_6 = \frac{\sqrt{2}}{2} l_6 = \frac{\sqrt{2}}{2} \times \frac{2\sqrt{3}}{3} a_6 = \frac{\sqrt{6}}{3} a_6$$

Radio " $k_6$ " de la circunferencia inscrita al polígono regular de una cara del mismo (apotema).

Se deduce de la fórmula 16, lám. 2

$$k_6 = \frac{1}{2} l_6 = \frac{1}{2} \times \frac{2\sqrt{3}}{3} a_6 = \frac{\sqrt{3}}{3} a_6$$

Ángulo rectilíneo " $2\varphi_6$ " del diedro del mismo.

Se deduce de la fórmula 15, lám. 2

$$\text{sen } \varphi_6 = \frac{\sqrt{2}}{2} \quad 2\varphi_6 = 90^\circ$$

Tomando como base los valores anteriores, deduciremos los siguientes del poliedro derivado.

Ángulo rectilíneo " $2\alpha_6$ " del diedro formado por dos ca-

Let  $x$  and  $y$  be the two numbers.

From the first condition, we have

$$x + y = 10 \quad (1)$$

From the second condition, we have

$$x - y = 2 \quad (2)$$

Adding equations (1) and (2), we get

$$2x = 12 \Rightarrow x = 6$$

Substituting  $x = 6$  in equation (1), we get

$$6 + y = 10 \Rightarrow y = 4$$

Thus, the two numbers are 6 and 4.

$$x = 6, y = 4$$

Verification:  $6 + 4 = 10$  and  $6 - 4 = 2$

∴ The two numbers are 6 and 4.

$$x = 6, y = 4$$

∴ The two numbers are 6 and 4.

∴ The two numbers are 6 and 4.

∴ The two numbers are 6 and 4.



ras contiguas del poliedro derivado, en una arista del exaedro dado.

Se deduce de la fórmula general [4] (ver lám. 25), sustituyendo en ella los valores particulares de este caso,

$$\frac{1}{2} \alpha_6 = \frac{a_6 \cdot k_6}{(k_6)^2 - a_6 c_6 + (c_6)^2} = \frac{a_6 \cdot \frac{\sqrt{3}}{3} a_6}{\left(\frac{\sqrt{3}}{3} a_6\right)^2 - a_6 \frac{\sqrt{3}}{3} a_6 + \left(\frac{\sqrt{3}}{3} a_6\right)^2} = 2\sqrt{3} + 3$$

$$\lg \frac{1}{2} \alpha_6 = \lg (2\sqrt{3} + 3) = \lg 6,4641016... \quad 0,8105082... = \lg \alpha_6$$

$$\alpha_6 = 81^\circ 12' 21,7''$$

$$2\alpha_6 = 162^\circ 24' 43,4''$$

El valor de  $\alpha_6 < 90^\circ$  nos demuestra la convexidad del poliedro derivado (ver lám. 25 "Consideraciones previas").

Altura "p" de una cara lateral de la pirámide recta formada en cada cara del exaedro dado (cara del poliedro derivado).

Se deduce de la fórmula [5] (ver lám. 25) sustituyendo en ella los valores particulares de este caso.

$$p = \sqrt{(a_6 - c_6)^2 + (k_6)^2} = \sqrt{\left(a_6 - \frac{\sqrt{3}}{3} a_6\right)^2 + \left(\frac{\sqrt{3}}{3} a_6\right)^2} = \sqrt{\frac{5-2\sqrt{3}}{3}} a_6$$

$$\text{Desarrollo del cálculo anterior: } p = \sqrt{\left(a_6 - \frac{\sqrt{3}}{3} a_6\right)^2 + \left(\frac{\sqrt{3}}{3} a_6\right)^2} =$$

$$= \sqrt{\left(a_6\right)^2 \left(1 - \frac{\sqrt{3}}{3}\right)^2 + \frac{3}{9} (a_6)^2} = \sqrt{\left(\frac{3-\sqrt{3}}{3}\right)^2 + \frac{3}{9}} a_6 = \sqrt{\frac{9+3-6\sqrt{3}}{9} + \frac{3}{9}} a_6 = \sqrt{\frac{5-2\sqrt{3}}{3}} a_6$$

No. 10	Date: 10/10/1910	Page: 1
<p>           The following is a list of the names of the persons who have been elected to the office of the Board of Directors of the City of New York for the year 1910.         </p>		
<p>           The names of the persons who have been elected to the office of the Board of Directors of the City of New York for the year 1910 are as follows:         </p>		
<p>           The names of the persons who have been elected to the office of the Board of Directors of the City of New York for the year 1910 are as follows:         </p>		
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Arista lateral "q" de la pirámide recta regular, o lado igual del triángulo isósceles de una cara del poliedro derivado.

Se deduce de la fórmula general [6] (ver lám. 25) sustituyendo en ella los valores particulares de este caso.

$$q = \sqrt{(a_6 - c_6)^2 + (d_6)^2} = \sqrt{\left(a_6 - \frac{\sqrt{3}}{3} a_6\right)^2 + \left(\frac{\sqrt{6}}{3} a_6\right)^2} = \sqrt{\frac{6-2\sqrt{3}}{3}} a_6$$

Desarrollo del cálculo anterior:  $q = \sqrt{\left(a_6 - \frac{\sqrt{3}}{3} a_6\right)^2 + \left(\frac{\sqrt{6}}{3} a_6\right)^2} =$

$$= \sqrt{\left(1 - \frac{\sqrt{3}}{3}\right)^2 + \frac{6}{9}} a_6 = \sqrt{\left(\frac{3-\sqrt{3}}{3}\right)^2 + \frac{6}{9}} a_6 = \sqrt{\frac{9+3-6\sqrt{3}}{9} + \frac{6}{9}} a_6 =$$

$$= \sqrt{\frac{18-6\sqrt{3}}{9}} a_6 = \boxed{\sqrt{\frac{6-2\sqrt{3}}{3}} a_6}$$



Diagonal "t" que se obtiene al unir los extremos de dos lados consecutivos del polígono de una cara del coaedro dado.

Es la diagonal de un cuadrado, cuyo valor sea

$$t = \sqrt{2} \ell_6 = \sqrt{2} \times \frac{2\sqrt{3}}{3} a_6 = \frac{2\sqrt{6}}{3} a_6$$

Ángulo rectilíneo del diedro "2γ<sub>6</sub>" formado por dos caras laterales contiguas en las aristas de la pirámide recta.

I am writing to you to inform you that I have received your letter of the 15th of the month and I am very glad to hear from you.

I am well and hope you are the same. I am very busy at the moment but I will try to write to you more often.

I am very glad to hear from you and I hope you are well. I am very busy at the moment but I will try to write to you more often.

I am very glad to hear from you and I hope you are well. I am very busy at the moment but I will try to write to you more often.

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Se deduce de la fórmula general [7] (ver lám. 25),  
sustituyendo en ella los valores particulares de este caso.

$$\operatorname{sen} \gamma_6 = \frac{t_9}{2 l_6 p} = \frac{\frac{2\sqrt{6}}{3} a_6 \times \sqrt{\frac{6-2\sqrt{3}}{3}} a_6}{2 \times \frac{2\sqrt{3}}{3} a_6 \times \sqrt{\frac{5-2\sqrt{3}}{3}} a_6} = \sqrt{\frac{9+\sqrt{3}}{13}}$$

Desarrollo del cálculo anterior:  $\boxed{\operatorname{sen} \gamma} = \frac{\frac{2\sqrt{6}}{3} a_6 \times \sqrt{\frac{6-2\sqrt{3}}{3}} a_6}{2 \times \frac{2\sqrt{3}}{3} a_6 \times \sqrt{\frac{5-2\sqrt{3}}{3}} a_6} =$

$$= \frac{\sqrt{6}}{2\sqrt{3}} \times \sqrt{\frac{6-2\sqrt{3}}{3} \cdot \frac{5-2\sqrt{3}}{3}} = \frac{\sqrt{2}}{2} \sqrt{\frac{6-2\sqrt{3}}{5-2\sqrt{3}}} = \frac{\sqrt{2}}{2} \sqrt{\frac{(6-2\sqrt{3})(5+2\sqrt{3})}{13}} =$$

$$= \frac{\sqrt{2}}{2} \sqrt{\frac{30-10\sqrt{3}+10\sqrt{3}-12}{13}} = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 \times \frac{18+2\sqrt{3}}{13}} = \sqrt{\frac{1}{2} \times \frac{18+2\sqrt{3}}{13}} = \boxed{\sqrt{\frac{9+\sqrt{3}}{13}}}$$

$$\operatorname{sen} \gamma_6 = \sqrt{\frac{9+\sqrt{3}}{13}} = 0,9085936\dots$$

$$\lg. \operatorname{sen} \gamma_6 = 7,9583097$$

$$\gamma_6 = 65^\circ 18' 42,2''$$

$$2\gamma_6 = 130^\circ 37' 34,4''$$



Let  $f(x) = \frac{1}{x^2} = x^{-2}$ . Then  $f'(x) = -2x^{-3} = -\frac{2}{x^3}$ .  
 The derivative of  $f(x)$  is  $-\frac{2}{x^3}$ .

$$\frac{d}{dx} \left( \frac{1}{x^2} \right) = \frac{d}{dx} (x^{-2}) = -2x^{-3} = -\frac{2}{x^3}$$

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Radio "b," de la esfera tangente a las aristas del poliedro regular dado.

Se deduce de la fórmula 12, lám. 2

$$b_1 = b_6 = \frac{\sqrt{2}}{2} l_6 = \frac{\sqrt{2}}{2} \times \frac{2\sqrt{3}}{3} a_6 = \frac{\sqrt{6}}{3} a_6$$

Ángulo diedro "β<sub>6</sub>" formado por una cara lateral de la pirámide y su base.

Se deduce de la fórmula general [8] (ver condiciones previas, lám. 25) sustituyendo en ella los valores particulares de este caso.

$$\text{sen } \beta_6 = \frac{a_6 - c_6}{p} = \frac{a_6 - \frac{\sqrt{3}}{3} a_6}{\sqrt{\frac{5-2\sqrt{3}}{3}} a_6} = \sqrt{\frac{8-2\sqrt{3}}{13}}$$

Desarrollo del cálculo anterior:

$$\boxed{\text{sen } \beta_6} = \frac{a_6 - \frac{\sqrt{3}}{3} a_6}{\sqrt{\frac{5-2\sqrt{3}}{3}} a_6} =$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{\sqrt{\frac{5-2\sqrt{3}}{3}}} = \frac{3 - \sqrt{3}}{3 \times \sqrt{\frac{5-2\sqrt{3}}{3}}} = \frac{(3-\sqrt{3}) \times \sqrt{\frac{5-2\sqrt{3}}{3}}}{3 \times \frac{5-2\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{5-2\sqrt{3}} \times \sqrt{\frac{5-2\sqrt{3}}{3}} =$$

$$= \frac{(3-\sqrt{3})(5-2\sqrt{3})}{13} \times \sqrt{\frac{5-2\sqrt{3}}{3}} = \frac{15-5\sqrt{3}+6\sqrt{3}-6}{13} \times \sqrt{\frac{5-2\sqrt{3}}{3}} =$$

$$= \frac{9+\sqrt{3}}{13} \times \sqrt{\frac{5-2\sqrt{3}}{3}} = \sqrt{\frac{(9+\sqrt{3})^2 (5-2\sqrt{3})}{13^2 \times 3}} = \sqrt{\frac{(81+3+18\sqrt{3})(5-2\sqrt{3})}{13^2 \times 3}} =$$

The first part of the problem is to find the value of the function  $f(x)$  at  $x = 1$ .

$$f(1) = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 3$$

Next, we need to find the value of the function  $f(x)$  at  $x = 2$ .

We can do this by substituting  $x = 2$  into the function  $f(x)$ .

$$f(2) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

Now, we need to find the value of the function  $f(x)$  at  $x = 3$ .

$$f(3) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

Finally, we need to find the value of the function  $f(x)$  at  $x = 4$ .

$$f(4) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

$$= \sqrt{\frac{(84 + 18\sqrt{3})(5 - 2\sqrt{3})}{13^2 \times 3}} = \sqrt{\frac{6 \times (14 + 3\sqrt{3})(5 - 2\sqrt{3})}{13^2 \times 3}} = \sqrt{\frac{2 \times (70 - 22\sqrt{3} + 15\sqrt{3} - 12)}{13^2}}$$

$$= \sqrt{\frac{2 \times (58 - 13\sqrt{3})}{13^2}} = \sqrt{\frac{2 \times (4 - \sqrt{3})}{13}} = \boxed{\sqrt{\frac{8 - 2\sqrt{3}}{13}}}$$

El valor numérico de  $\beta_6$ , expresado en grados sexagesimales, es el siguiente:

$$\text{sen } \beta_6 = \sqrt{\frac{8 - 2\sqrt{3}}{13}} = 0,5906905 \quad \text{y} \quad \text{sen } \beta_6 = 7,7713600$$

$$\beta_6 = 36^\circ 12' 21,7''$$

debiendo verificarse como comprobación (ver fórm. [71], consideraciones previas, lám. 25) que

$$\alpha_6 = \varphi_6 + \beta_6 = 45^\circ + 36^\circ 12' 21,7'' = 81^\circ 12' 21,7''$$

valor coincidente al ya obtenido de  $\alpha_6$ .

Radio "b<sub>2</sub>" de la esfera tangente a las aristas laterales de las pirámides rectas cuyas bases son caras del exaedro regular dado.

Se deduce de la fórmula general [9] (ver consideraciones previas, lám. 25), sustituyendo en ella los valores particulares de este caso.

$$b_2 = \sqrt{(a_6)^2 - \frac{9}{4}} = \sqrt{(a_6)^2 - \left(\sqrt{\frac{6 - 2\sqrt{3}}{3}} a_6\right)^2 : 4} = \sqrt{\frac{3 + \sqrt{3}}{6}} a_6$$

1. The first part of the paper is devoted to a general discussion of the problem.

$$\left( \frac{1}{2} \right)^n = \frac{1}{2^n}$$

2. In the second part, we consider the case of a finite number of variables.

3. The third part is devoted to the study of the properties of the function.

4. In the fourth part, we consider the case of a continuous function.

5. The fifth part is devoted to the study of the properties of the function.

6. In the sixth part, we consider the case of a continuous function.

7. The seventh part is devoted to the study of the properties of the function.

$$\frac{1}{2^n} = \frac{1}{2^n}$$



Desarrollo del cálculo anterior:  $\boxed{b_2} = \sqrt{(a_6)^2 - \left(\frac{\sqrt{6-2\sqrt{3}}}{3} a_6\right)^2} : 4 =$

$$= \sqrt{(a_6)^2 - \frac{6-2\sqrt{3}}{3} : 4 (a_6)^2} = \sqrt{1 - \frac{3-\sqrt{3}}{6}} a_6 = \sqrt{\frac{6-3+\sqrt{3}}{6}} a_6 = \boxed{\sqrt{\frac{3+\sqrt{3}}{6}} a_6}$$

Radio "C<sub>1</sub>" de la esfera inscrita en el poliedro derivado

Se deduce de la fórmula general [10] (ver consideraciones previas, lám. 25), sustituyendo en ella los valores particulares de este caso.

$$C_1 = b_1 \operatorname{sen} \alpha_6 \quad \text{siendo} \quad b_1 = \frac{\sqrt{6}}{3} a_6 \quad \text{y} \quad \frac{1}{2} \alpha_6 = 2\sqrt{3} + 3$$

$$\text{pero} \quad \operatorname{sen} \alpha_6 = \frac{\frac{1}{2} \alpha_6}{\sqrt{1 + \left(\frac{1}{2} \alpha_6\right)^2}} = \frac{2\sqrt{3} + 3}{\sqrt{1 + (2\sqrt{3} + 3)^2}} \quad \text{de donde}$$

$$C_1 = b_1 \operatorname{sen} \alpha_6 = \frac{\sqrt{6}}{3} a_6 \times \frac{2\sqrt{3} + 3}{\sqrt{1 + (2\sqrt{3} + 3)^2}} = \sqrt{\frac{5 + 2\sqrt{3}}{13}} a_6$$

$$\text{Desarrollo del cálculo anterior: } C_1 = \frac{\sqrt{6}}{3} a_6 \times \frac{2\sqrt{3} + 3}{\sqrt{1 + (2\sqrt{3} + 3)^2}} =$$

$$= \frac{\sqrt{6}}{3} \times \frac{2\sqrt{3} + 3}{\sqrt{1 + 12 + 9 + 12\sqrt{3}}} a_6 = \frac{\sqrt{6}}{3} \times \frac{2\sqrt{3} + 3}{\sqrt{2 \times (11 + 6\sqrt{3})}} a_6 =$$

$$= \frac{\sqrt{6} \times \sqrt{2 \times (11 + 6\sqrt{3})}}{3} \times \frac{2\sqrt{3} + 3}{2 \times (11 + 6\sqrt{3})} a_6 = \frac{\sqrt{12 (11 + 6\sqrt{3})}}{3} \times \frac{(2\sqrt{3} + 3)(11 - 6\sqrt{3})}{2 \times (121 - 108)} a_6 =$$

$$= \frac{\sqrt{12 (11 + 6\sqrt{3})}}{3} \times \frac{(2\sqrt{3} + 3)(11 - 6\sqrt{3})}{2 \times 13} a_6 = \frac{\sqrt{12 (11 + 6\sqrt{3})} (11 - 6\sqrt{3})^2}{3} \times \frac{2\sqrt{3} + 3}{2 \times 13} a_6$$

The Laplace transform is a powerful tool for solving differential equations. It converts a differential equation in the time domain into an algebraic equation in the frequency domain.

The Laplace transform of a function  $f(t)$  is defined as:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

where  $s$  is a complex number. The inverse Laplace transform is defined as:

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} F(s) e^{st} ds$$

The Laplace transform is linear, meaning that:

$$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$$

The Laplace transform of a derivative is given by:

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

The Laplace transform of an integral is given by:

$$\mathcal{L}\left\{\int_0^t f(\tau) d\tau\right\} = \frac{F(s)}{s}$$

The Laplace transform of a product of two functions is given by:

$$\mathcal{L}\{f(t)g(t)\} = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} F(s)G(s) ds$$

The Laplace transform of a convolution is given by:

$$\mathcal{L}\{f(t) * g(t)\} = F(s)G(s)$$

$$\rightarrow \frac{\sqrt{3 \times (11 - 6\sqrt{3}) \times (121 - 108)} \times (2\sqrt{3} + 3)}{3 \times 13} a_6 = \frac{\sqrt{3 \times 13 \times (11 - 6\sqrt{3}) (2\sqrt{3} + 3)^2}}{3 \times 13} a_6 =$$

$$= \frac{\sqrt{3 \times 13 \times (11 - 6\sqrt{3}) (12 + 9 + 12\sqrt{3})}}{3 \times 13} a_6 = \frac{\sqrt{3 \times 13 \times (11 - 6\sqrt{3}) (21 + 12\sqrt{3})}}{3 \times 13} a_6 =$$

$$= \frac{\sqrt{3 \times 3 \times 13 \times (11 - 6\sqrt{3}) (7 + 4\sqrt{3})}}{3 \times 13} a_6 = \frac{\sqrt{13 \times (77 - 42\sqrt{3} + 44\sqrt{3} - 72)}}{13} a_6 =$$

$$= \frac{\sqrt{13 \times (5 + 2\sqrt{3})}}{13} a_6 = \sqrt{\frac{13 \times (5 + 2\sqrt{3})}{13^2}} a_6 = \boxed{\sqrt{\frac{5 + 2\sqrt{3}}{13}} a_6}$$

Este mismo valor se puede deducir de la fórmula equivalente [10'] (lámina 25), en la que

$$\boxed{C_7} = b_2 \operatorname{sen} \gamma_6 = \sqrt{\frac{3 + \sqrt{3}}{6}} a_6 \times \sqrt{\frac{9 + \sqrt{3}}{13}} = \sqrt{\frac{(3 + \sqrt{3})(9 + \sqrt{3})}{6 \times 13}} a_6 =$$

$$= \sqrt{\frac{27 + 9\sqrt{3} + 3\sqrt{3} + 3}{6 \times 13}} a_6 = \sqrt{\frac{30 + 12\sqrt{3}}{6 \times 13}} a_6 = \boxed{\sqrt{\frac{5 + 2\sqrt{3}}{13}} a_6}$$

### Área lateral "S" del poliedro derivado

Se obtiene como suma de las áreas laterales de las seis pirámides rectas de base cuadrada, cuyas caras son triángulos isósceles de base " $b_6$ " y altura " $p$ ", ya determinados.

(sigue en hoja 11)

Let  $f(x) = \frac{1}{x^2}$  and  $g(x) = \frac{1}{x^3}$ . Then  $f(x)g(x) = \frac{1}{x^5}$ .

$$\frac{d}{dx} \left( \frac{1}{x^5} \right) = -\frac{5}{x^6} = -\frac{5}{x^2} \cdot \frac{1}{x^4} = -5f(x)g(x).$$

$$\frac{d}{dx} \left( \frac{1}{x^2} \cdot \frac{1}{x^3} \right) = -\frac{5}{x^6} = -5 \cdot \frac{1}{x^2} \cdot \frac{1}{x^4} = -5f(x)g(x).$$

$$\left( \frac{d}{dx} \frac{1}{x^2} \right) \cdot \frac{1}{x^3} + \frac{1}{x^2} \cdot \left( \frac{d}{dx} \frac{1}{x^3} \right) = -\frac{2}{x^3} \cdot \frac{1}{x^3} + \frac{1}{x^2} \cdot \left( -\frac{3}{x^4} \right) = -\frac{2}{x^6} - \frac{3}{x^6} = -\frac{5}{x^6} = -5f(x)g(x).$$

Therefore,  $\frac{d}{dx} (f(x)g(x)) = -5f(x)g(x)$ .

$$\frac{d}{dx} \left( \frac{1}{x^2} \cdot \frac{1}{x^3} \right) = -\frac{5}{x^6} = -5 \cdot \frac{1}{x^2} \cdot \frac{1}{x^4} = -5f(x)g(x).$$

$$\left( \frac{d}{dx} \frac{1}{x^2} \right) \cdot \frac{1}{x^3} + \frac{1}{x^2} \cdot \left( \frac{d}{dx} \frac{1}{x^3} \right) = -\frac{2}{x^3} \cdot \frac{1}{x^3} + \frac{1}{x^2} \cdot \left( -\frac{3}{x^4} \right) = -\frac{2}{x^6} - \frac{3}{x^6} = -\frac{5}{x^6} = -5f(x)g(x).$$

Therefore,  $\frac{d}{dx} (f(x)g(x)) = -5f(x)g(x)$ .

Let  $f(x) = \frac{1}{x^2}$  and  $g(x) = \frac{1}{x^3}$ . Then  $f(x)g(x) = \frac{1}{x^5}$ .  
 $\frac{d}{dx} \left( \frac{1}{x^5} \right) = -\frac{5}{x^6} = -\frac{5}{x^2} \cdot \frac{1}{x^4} = -5f(x)g(x)$   
 $\frac{d}{dx} \left( \frac{1}{x^2} \cdot \frac{1}{x^3} \right) = -\frac{5}{x^6} = -5 \cdot \frac{1}{x^2} \cdot \frac{1}{x^4} = -5f(x)g(x)$   
 $\left( \frac{d}{dx} \frac{1}{x^2} \right) \cdot \frac{1}{x^3} + \frac{1}{x^2} \cdot \left( \frac{d}{dx} \frac{1}{x^3} \right) = -\frac{2}{x^3} \cdot \frac{1}{x^3} + \frac{1}{x^2} \cdot \left( -\frac{3}{x^4} \right) = -\frac{2}{x^6} - \frac{3}{x^6} = -\frac{5}{x^6} = -5f(x)g(x)$

Therefore,  $\frac{d}{dx} (f(x)g(x)) = -5f(x)g(x)$ .

$$S = 6 \times 4 \times \frac{l_c \cdot p}{2} = 6 \times 4 \times \frac{\frac{2\sqrt{3}}{3} a_6 \times \sqrt{\frac{5-2\sqrt{3}}{3}} a_6}{2} = 8 \sqrt{5-2\sqrt{3}} (a_6)^2$$

Desarrollo del cálculo anterior: 
$$S = 6 \times 4 \times \frac{\frac{2\sqrt{3}}{3} a_6 \times \sqrt{\frac{5-2\sqrt{3}}{3}} a_6}{2} =$$

$$= \frac{6 \times 4 \times 2}{3 \times 2} \sqrt{3} \sqrt{\frac{5-2\sqrt{3}}{3}} (a_6)^2 = 8 \sqrt{\frac{5-2\sqrt{3}}{3} \times (\sqrt{3})^2} a_6^2 = 8 \sqrt{5-2\sqrt{3}} (a_6)^2$$

Volumen "V" del poliedro derivado

Se obtiene como suma del volumen del exaedro dado y de las seis pirámides de sus caras.

$$V = V_6 + 6 \times \frac{S_4 \times h}{3}$$

siendo " $S_4$ " el área de una cara del exaedro, y " $h$ " la altura de la pirámide.

Para obtener  $V_6$  en función de  $a_6$ , ver lám. 2, fórm. 19 y 11, que nos dan

$$V_6 = (l_c)^3 = \left(\frac{2}{\sqrt{3}} a_6\right)^3 = \frac{8}{3\sqrt{3}} (a_6)^3 = \frac{8\sqrt{3}}{9} (a_6)^3$$

Por otra parte tendremos que

$$S_4 = (l_c)^2 = \left(\frac{2}{\sqrt{3}} a_6\right)^2 = \frac{4}{3} (a_6)^2 \quad \text{y también que (ver lám. 2,$$

fórm. 11 y 13)

$$h = a_6 - c_6 = \frac{\sqrt{3}}{2} l_c - \frac{1}{2} l_c = \frac{\sqrt{3}-1}{2} \times l_c = \frac{\sqrt{3}-1}{2} \times \frac{2}{\sqrt{3}} a_6 = \frac{\sqrt{3}-1}{\sqrt{3}} a_6 = \frac{3-\sqrt{3}}{3} a_6$$



1. The first part of the problem is to find the value of the function  $f(x)$  at the point  $x = 1$ .

2. The second part is to find the value of the function  $f(x)$  at the point  $x = 2$ .

3. The third part is to find the value of the function  $f(x)$  at the point  $x = 3$ .

4. The fourth part is to find the value of the function  $f(x)$  at the point  $x = 4$ .

5. The fifth part is to find the value of the function  $f(x)$  at the point  $x = 5$ .

6. The sixth part is to find the value of the function  $f(x)$  at the point  $x = 6$ .

7. The seventh part is to find the value of the function  $f(x)$  at the point  $x = 7$ .

8. The eighth part is to find the value of the function  $f(x)$  at the point  $x = 8$ .

9. The ninth part is to find the value of the function  $f(x)$  at the point  $x = 9$ .

10. The tenth part is to find the value of the function  $f(x)$  at the point  $x = 10$ .

11. The eleventh part is to find the value of the function  $f(x)$  at the point  $x = 11$ .

12. The twelfth part is to find the value of the function  $f(x)$  at the point  $x = 12$ .

y finalmente:

$$V = V_6 + 6 \times \frac{S_4 \cdot h}{3} = \frac{8\sqrt{3}}{9} (a_6)^3 + 6 \times \frac{\frac{4}{3} (a_6)^2 \times \frac{3-\sqrt{3}}{3} a_6}{3} = \frac{8}{3} (a_6)^3$$

Desarrollo del cálculo anterior:  $\boxed{V} = \frac{8\sqrt{3}}{9} (a_6)^3 + 6 \times \frac{\frac{4}{3} (a_6)^2 \times \frac{3-\sqrt{3}}{3} a_6}{3} =$

$$= \left( \frac{8\sqrt{3}}{9} + \frac{6 \times 4 \times (3-\sqrt{3})}{9 \times 3} \right) (a_6)^3 = \left( \frac{8\sqrt{3}}{9} + \frac{8(3-\sqrt{3})}{9} \right) (a_6)^3 =$$

$$= \frac{8}{9} (\sqrt{3} + 3 - \sqrt{3}) (a_6)^3 = \boxed{\frac{8}{3} (a_6)^3}$$

(sigue en h. 13)

1.  $\frac{1}{x^2} = x^{-2}$  ;  $\frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$

2.  $\frac{d}{dx} \ln(x) = \frac{1}{x}$  ;  $\frac{d}{dx} \ln(x^2) = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$

3.  $\frac{d}{dx} e^x = e^x$  ;  $\frac{d}{dx} e^{2x} = e^{2x} \cdot 2 = 2e^{2x}$

4.  $\frac{d}{dx} \sin(x) = \cos(x)$  ;  $\frac{d}{dx} \sin(2x) = \cos(2x) \cdot 2 = 2\cos(2x)$

5.  $\frac{d}{dx} \cos(x) = -\sin(x)$  ;  $\frac{d}{dx} \cos(2x) = -\sin(2x) \cdot 2 = -2\sin(2x)$

En el cuadro sinóptico que damos a continuación, resumimos los resultados anteriores.

## CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
<sup>252</sup> $l_6$	$\frac{2\sqrt{3}}{3} a_6$	1. 15 47 01... $a_6$
<sup>253</sup> $b_1$	$\frac{\sqrt{6}}{3} a_6$	0. 81 64 97... $a_6$
<sup>254</sup> $b_2$	$\sqrt{\frac{3+\sqrt{3}}{6}} a_6$	0. 88 80 74... $a_6$
<sup>255</sup> $c_6$	$\frac{\sqrt{3}}{3} a_6$	0. 57 73 50... $a_6$
<sup>256</sup> $c_1$	$\sqrt{\frac{5+2\sqrt{3}}{13}} a_6$	0. 80 68 98... $a_6$
<sup>257</sup> $d_6$	$\frac{\sqrt{6}}{3} a_6$	0. 81 64 97... $a_6$
<sup>258</sup> $k_6$	$\frac{\sqrt{3}}{3} a_6$	0. 57 73 50... $a_6$
<sup>259</sup> $2\psi_6$	$\text{sen } \psi_6 = \frac{\sqrt{2}}{2}$	$\text{sen } \psi_6 = 0.707107$ $2\psi_6 = 90^\circ$
<sup>260</sup> $2\alpha_6$	$\text{tg } \alpha_6 = 2\sqrt{3} + 3$	$\text{tg } \alpha_6 = 0.810508$ $2\alpha_6 = 162^\circ 24' 43.4''$
<sup>261</sup> $2\gamma_6$	$\text{sen } \gamma_6 = \sqrt{\frac{9+\sqrt{3}}{13}}$	$\text{sen } \gamma_6 = 0.908594$ $2\gamma_6 = 130^\circ 37' 24.4''$
<sup>262</sup> $\beta_6$	$\text{sen } \beta_6 = \sqrt{\frac{8-2\sqrt{3}}{13}}$	$\text{sen } \beta_6 = 0.590691$ $\beta_6 = 36^\circ 12' 27.7''$
<sup>263</sup> $p$	$\sqrt{\frac{5-2\sqrt{3}}{3}} a_6$	0. 71 55 18... $a_6$
<sup>264</sup> $q$	$\sqrt{\frac{6-2\sqrt{3}}{3}} a_6$	0. 91 94 02... $a_6$
<sup>265</sup> $t$	$\frac{2\sqrt{6}}{3} a_6$	1. 63 29 93... $a_6$
<sup>266</sup> $S$	$8\sqrt{5-2\sqrt{3}} (a_6)^2$	9. 91 45 09... $(a_6)^2$
<sup>267</sup> $V$	$\frac{8}{3} (a_6)^3$	2. 66 66 67... $(a_6)^3$



The following table shows the results of the experiment conducted on the 10th of May 1900. The results are given in the form of a table, and the data is as follows:

Table showing the results of the experiment.

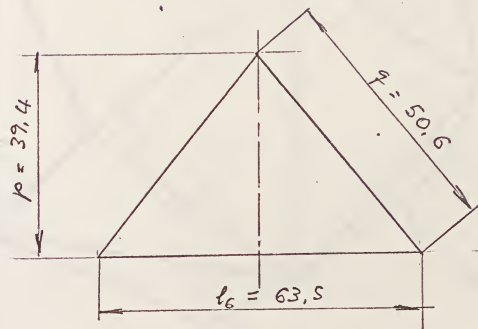
Time	Temperature	Pressure
10.00	20.0	101.0
10.10	20.5	101.5
10.20	21.0	102.0
10.30	21.5	102.5
10.40	22.0	103.0
10.50	22.5	103.5
11.00	23.0	104.0
11.10	23.5	104.5
11.20	24.0	105.0
11.30	24.5	105.5
11.40	25.0	106.0
11.50	25.5	106.5
12.00	26.0	107.0
12.10	26.5	107.5
12.20	27.0	108.0
12.30	27.5	108.5
12.40	28.0	109.0
12.50	28.5	109.5
13.00	29.0	110.0
13.10	29.5	110.5
13.20	30.0	111.0
13.30	30.5	111.5
13.40	31.0	112.0
13.50	31.5	112.5
14.00	32.0	113.0
14.10	32.5	113.5
14.20	33.0	114.0
14.30	33.5	114.5
14.40	34.0	115.0
14.50	34.5	115.5
15.00	35.0	116.0
15.10	35.5	116.5
15.20	36.0	117.0
15.30	36.5	117.5
15.40	37.0	118.0
15.50	37.5	118.5
16.00	38.0	119.0
16.10	38.5	119.5
16.20	39.0	120.0
16.30	39.5	120.5
16.40	40.0	121.0
16.50	40.5	121.5
17.00	41.0	122.0
17.10	41.5	122.5
17.20	42.0	123.0
17.30	42.5	123.5
17.40	43.0	124.0
17.50	43.5	124.5
18.00	44.0	125.0
18.10	44.5	125.5
18.20	45.0	126.0
18.30	45.5	126.5
18.40	46.0	127.0
18.50	46.5	127.5
19.00	47.0	128.0
19.10	47.5	128.5
19.20	48.0	129.0
19.30	48.5	129.5
19.40	49.0	130.0
19.50	49.5	130.5
20.00	50.0	131.0
20.10	50.5	131.5
20.20	51.0	132.0
20.30	51.5	132.5
20.40	52.0	133.0
20.50	52.5	133.5
21.00	53.0	134.0
21.10	53.5	134.5
21.20	54.0	135.0
21.30	54.5	135.5
21.40	55.0	136.0
21.50	55.5	136.5
22.00	56.0	137.0
22.10	56.5	137.5
22.20	57.0	138.0
22.30	57.5	138.5
22.40	58.0	139.0
22.50	58.5	139.5
23.00	59.0	140.0
23.10	59.5	140.5
23.20	60.0	141.0
23.30	60.5	141.5
23.40	61.0	142.0
23.50	61.5	142.5
24.00	62.0	143.0
24.10	62.5	143.5
24.20	63.0	144.0
24.30	63.5	144.5
24.40	64.0	145.0
24.50	64.5	145.5
25.00	65.0	146.0
25.10	65.5	146.5
25.20	66.0	147.0
25.30	66.5	147.5
25.40	67.0	148.0
25.50	67.5	148.5
26.00	68.0	149.0
26.10	68.5	149.5
26.20	69.0	150.0
26.30	69.5	150.5
26.40	70.0	151.0
26.50	70.5	151.5
27.00	71.0	152.0
27.10	71.5	152.5
27.20	72.0	153.0
27.30	72.5	153.5
27.40	73.0	154.0
27.50	73.5	154.5
28.00	74.0	155.0
28.10	74.5	155.5
28.20	75.0	156.0
28.30	75.5	156.5
28.40	76.0	157.0
28.50	76.5	157.5
29.00	77.0	158.0
29.10	77.5	158.5
29.20	78.0	159.0
29.30	78.5	159.5
29.40	79.0	160.0
29.50	79.5	160.5
30.00	80.0	161.0
30.10	80.5	161.5
30.20	81.0	162.0
30.30	81.5	162.5
30.40	82.0	163.0
30.50	82.5	163.5
31.00	83.0	164.0
31.10	83.5	164.5
31.20	84.0	165.0
31.30	84.5	165.5
31.40	85.0	166.0
31.50	85.5	166.5
32.00	86.0	167.0
32.10	86.5	167.5
32.20	87.0	168.0
32.30	87.5	168.5
32.40	88.0	169.0
32.50	88.5	169.5
33.00	89.0	170.0
33.10	89.5	170.5
33.20	90.0	171.0
33.30	90.5	171.5
33.40	91.0	172.0
33.50	91.5	172.5
34.00	92.0	173.0
34.10	92.5	173.5
34.20	93.0	174.0
34.30	93.5	174.5
34.40	94.0	175.0
34.50	94.5	175.5
35.00	95.0	176.0
35.10	95.5	176.5
35.20	96.0	177.0
35.30	96.5	177.5
35.40	97.0	178.0
35.50	97.5	178.5
36.00	98.0	179.0
36.10	98.5	179.5
36.20	99.0	180.0
36.30	99.5	180.5
36.40	100.0	181.0
36.50	100.5	181.5
37.00	101.0	182.0
37.10	101.5	182.5
37.20	102.0	183.0
37.30	102.5	183.5
37.40	103.0	184.0
37.50	103.5	184.5
38.00	104.0	185.0
38.10	104.5	185.5
38.20	105.0	186.0
38.30	105.5	186.5
38.40	106.0	187.0
38.50	106.5	187.5
39.00	107.0	188.0
39.10	107.5	188.5
39.20	108.0	189.0
39.30	108.5	189.5
39.40	109.0	190.0
39.50	109.5	190.5
40.00	110.0	191.0
40.10	110.5	191.5
40.20	111.0	192.0
40.30	111.5	192.5
40.40	112.0	193.0
40.50	112.5	193.5
41.00	113.0	194.0
41.10	113.5	194.5
41.20	114.0	195.0
41.30	114.5	195.5
41.40	115.0	196.0
41.50	115.5	196.5
42.00	116.0	197.0
42.10	116.5	197.5
42.20	117.0	198.0
42.30	117.5	198.5
42.40	118.0	199.0
42.50	118.5	199.5
43.00	119.0	200.0
43.10	119.5	200.5
43.20	120.0	201.0
43.30	120.5	201.5
43.40	121.0	202.0
43.50	121.5	202.5
44.00	122.0	203.0
44.10	122.5	203.5
44.20	123.0	204.0
44.30	123.5	204.5
44.40	124.0	205.0
44.50	124.5	205.5
45.00	125.0	206.0
45.10	125.5	206.5
45.20	126.0	207.0
45.30	126.5	207.5
45.40	127.0	208.0
45.50	127.5	208.5
46.00	128.0	209.0
46.10	128.5	209.5
46.20	129.0	210.0
46.30	129.5	210.5
46.40	130.0	211.0
46.50	130.5	211.5
47.00	131.0	212.0
47.10	131.5	212.5
47.20	132.0	213.0
47.30	132.5	213.5
47.40	133.0	214.0
47.50	133.5	214.5
48.00	134.0	215.0
48.10	134.5	215.5
48.20	135.0	216.0
48.30	135.5	216.5
48.40	136.0	217.0
48.50	136.5	217.5
49.00	137.0	218.0
49.10	137.5	218.5
49.20	138.0	219.0
49.30	138.5	219.5
49.40	139.0	220.0
49.50	139.5	220.5
50.00	140.0	221.0
50.10	140.5	221.5
50.20	141.0	222.0
50.30	141.5	222.5
50.40	142.0	223.0
50.50	142.5	223.5
51.00	143.0	224.0
51.10	143.5	224.5
51.20	144.0	225.0
51.30	144.5	225.5
51.40	145.0	226.0
51.50	145.5	226.5
52.00	146.0	227.0
52.10	146.5	227.5
52.20	147.0	228.0
52.30	147.5	228.5
52.40	148.0	229.0
52.50	148.5	229.5
53.00	149.0	230.0
53.10	149.5	230.5
53.20	150.0	231.0
53.30	150.5	231.5
53.40	151.0	232.0
53.50	151.5	232.5
54.00	152.0	233.0
54.10	152.5	233.5
54.20	153.0	234.0
54.30	153.5	234.5
54.40	154.0	235.0
54.50	154.5	235.5
55.00	155.0	236.0
55.10	155.5	236.5
55.20	156.0	237.0
55.30	156.5	237.5
55.40	157.0	238.0
55.50	157.5	238.5
56.00	158.0	239.0
56.10	158.5	239.5
56.20	159.0	240.0
56.30	159.5	240.5
56.40	160.0	241.0
56.50	160.5	241.5
57.00	161.0	242.0
57.10	161.5	242.5
57.20	162.0	243.0
57.30	162.5	243.5
57.40	163.0	244.0
57.50	163.5	244.5
58.00	164.0	245.0
58.10	164.5	245.5
58.20	165.0	246.0
58.30	165.5	246.5
58.40	166.0	247.0
58.50	166.5	247.5
59.00	167.0	248.0
59.10	167.5	248.5
59.20	168.0	249.0
59.30	168.5	249.5
59.40	169.0	250.0
59.50	169.5	250.5
60.00	170.0	251.0
60.10	170.5	251.5
60.20	171.0	252.0
60.30	171.5	252.5
60.40	172.0	253.0
60.50	172.5	253.5
61.00	173.0	254.0
61.10	173.5	254.5
61.20	174.0	255.0
61.30	174.5	255.5
61.40	175.0	256.0
61.50	175.5	256.5
62.00	176.0	257.0
62.10	176.5	257.5
62.20	177.0	258.0
62.30	177.5	258.5
62.40	178.0	259.0
62.50	178.5	259.5
63.00	179.0	260.0
63.10	179.5	260.5
63.20	180.0	261.0
63.30	180.5	261.5
63.40	181.0	262.0
63.50	181.5	262.5
64.00	182.0	263.0
64.10	182.5	263.5
64.20	183.0	264.0
64.30	183.5	264.5
64.40	184.0	265.0
64.50	184.5	265.5
65.00	185.0	266.0
65.10	185.5	266.5
65.20	186.0	267.0
65.30	186.5	267.5
65.40	187.0	268.0
65.50	187.5	268.5
66.00	188.0	269.0
66.10	188.5	269.5
66.20	189.0	270.0
66.30	189.5	270.5
66.40	190.0	271.0
66.50	19	



FIGURA CORPÓREA

Se obtiene por acoplamiento de 24 caras iguales en forma de triángulo isósceles, de base  $l_6 = 63,5$  mm y altura  $p = 39,4$  mm.; en este triángulo el lado igual  $q$ , tiene el valor  $q = 50,6$  mm (comprobación).

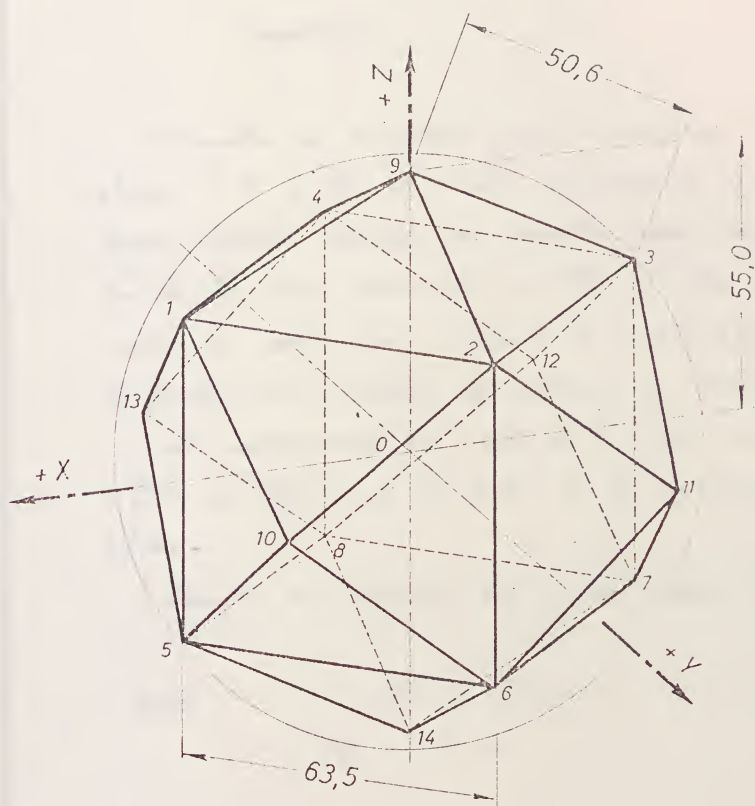
Para obtener este poliedro se formarán previamente 6 pirámides rectas de base cuadrada de lado " $l_6$ "; cuyas caras laterales son 4 triángulos (ver figura) acoplados por su lado " $q$ ".



The following is a list of the names of the persons who have been elected to the office of the President of the United States since the year 1789. The names are given in alphabetical order, and the year of election is given in parentheses.

George Washington (1789)  
 John Adams (1796)  
 Thomas Jefferson (1800)  
 James Madison (1808)  
 James Monroe (1816)  
 John Quincy Adams (1824)  
 Andrew Jackson (1828)  
 Martin Van Buren (1836)  
 William Henry Harrison (1840)  
 Franklin Pierce (1852)  
 Abraham Lincoln (1860)  
 Andrew Johnson (1865)  
 Ulysses S. Grant (1868)  
 Rutherford B. Hayes (1876)  
 James A. Garfield (1880)  
 Chester A. Arthur (1881)  
 Grover Cleveland (1885)  
 Benjamin Harrison (1889)  
 William McKinley (1896)  
 Theodore Roosevelt (1901)  
 William Howard Taft (1908)  
 Woodrow Wilson (1912)  
 Warren G. Harding (1921)  
 Calvin Coolidge (1923)  
 Herbert Hoover (1929)  
 Franklin D. Roosevelt (1932)  
 Harry S. Truman (1948)  
 Dwight D. Eisenhower (1952)  
 John F. Kennedy (1960)  
 Lyndon B. Johnson (1964)  
 Richard M. Nixon (1968)  
 Gerald R. Ford (1974)  
 Jimmy Carter (1976)  
 Ronald Reagan (1980)  
 George H. W. Bush (1988)  
 Bill Clinton (1992)  
 George W. Bush (2001)  
 Barack Obama (2008)  
 Donald Trump (2016)





*Pòliedro derivado del exaedro regular*



Figure 1. A cube with internal lines and axes.

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro derivado de un octaedro regular, obtenido al proyectar desde el centro de la esfera circunscrita a éste, y sobre ella, los centros de cada cara, uniendo a continuación estos puntos con los vértices del polígono de dicha cara.

Las coordenadas del centro de la esfera son:  $O(72, 72, 85)$  mm y el radio de la misma, de 55 mm.

Dibujar en formato A3V y a escala 1:1.

DATOS

$O(72, 72, 85)$  mm

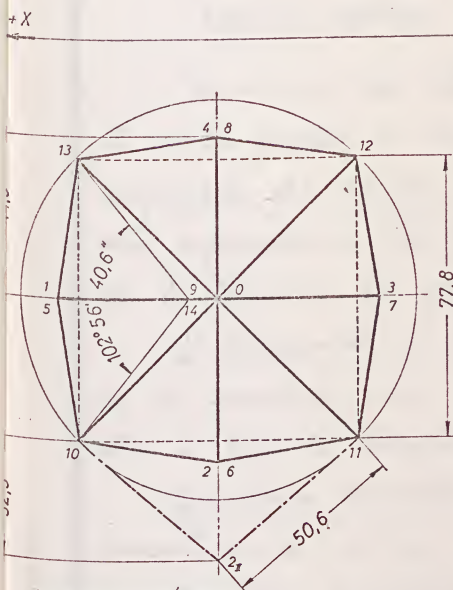
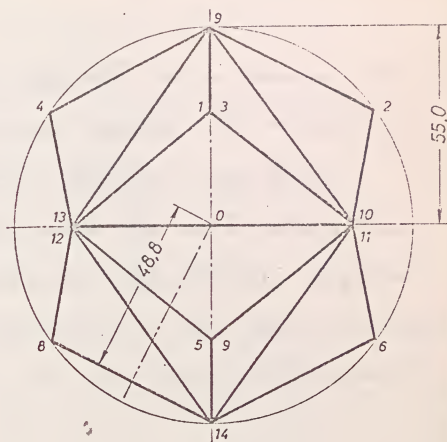
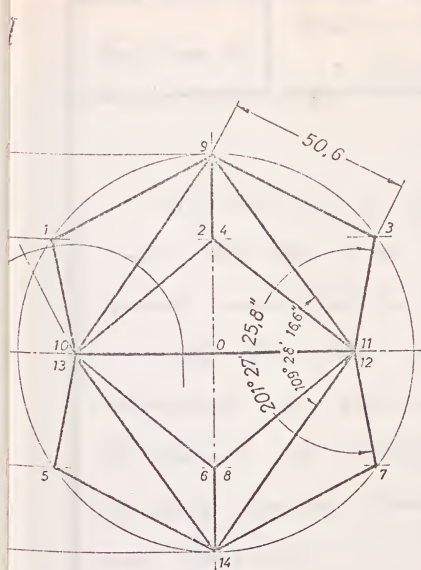
$a_g = 55$  mm.



The following is a summary of the results of the study conducted by the author and his associates. The study was designed to determine the effect of the use of the X-ray in the diagnosis of the disease known as "tuberculosis." The results of the study are as follows: The use of the X-ray in the diagnosis of tuberculosis is of great value. It is especially valuable in the early stages of the disease, when the symptoms are not yet pronounced. The X-ray can detect the disease at an early stage, when the patient is still in good health, and before the disease has become chronic. This is of great importance, as it allows the patient to receive treatment at an early stage, when the disease is still curable. The use of the X-ray in the diagnosis of tuberculosis is also of great value in the later stages of the disease, when the patient is already ill. The X-ray can detect the disease at a stage when the patient is still in good health, and before the disease has become chronic. This is of great importance, as it allows the patient to receive treatment at an early stage, when the disease is still curable.

The results of the study are as follows:

1. The use of the X-ray in the diagnosis of tuberculosis is of great value.  
 2. It is especially valuable in the early stages of the disease, when the symptoms are not yet pronounced.



#### NUMERACIÓN DE VÉRTICES

Octaedro regular..... 9 al 14

Proyecciones centros caras del mismo

(vértices del exaedro conjugado)..... 1 al 8

#### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III el poliedro derivado de un octaedro regular, obtenido al proyectar desde el centro de la esfera circunscrita a éste, y sobre ella, los centros de cada cara, uniendo a continuación estos puntos con los vértices del polígono de dicha cara.

Las coordenadas del centro de la esfera son: O (72, 72, 85) mm y el radio de la misma de 55 mm.

Dibujar en formato A3v y a escala 1:1.

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala	Poliedro derivado del octaedro regular				Lámina 27
1:1					Curso 12 -13



The first diagram shows a cube's net, which is a 2D representation of a 3D object. It consists of six squares arranged in a cross-like pattern. The second diagram shows a cube's net, which is a 2D representation of a 3D object. It consists of six squares arranged in a cross-like pattern. The third diagram shows a cube's net, which is a 2D representation of a 3D object. It consists of six squares arranged in a cross-like pattern.



Al estudiar el ejercicio propuesto en la lámina 25, hemos obtenido unas deducciones previas de carácter general, comunes a los cinco poliedros regulares.

Las fórmulas allí deducidas las aplicaremos sucesivamente en este caso particular del octaedro regular. El desarrollo del cálculo correspondiente a esta lámina, seguiremos pues aquellas directrices, a las que haremos las oportunas referencias.

#### PROCESO GRÁFICO

En el caso del poliedro derivado del octaedro regular, el proceso gráfico es inmediato, ya que sabemos que el conjugado del octaedro regular es el exaedro regular, y esta representación ha sido ya efectuada en el ejercicio de la lámina 23, cuyo proceso nos permite:

1º Representar el octaedro regular dado, de vértices 9 al 14, inscrito en una esfera de 55 mm de radio (estos vértices se han de corresponder con los 11 al 16 de la lám. 23).

2º Obtener los vértices del exaedro conjugado 1 al 8, inscrito en la misma esfera.

3º Unir los vértices 1 al 8 con los correspondientes de cada cara del octaedro dado.

Al terminar la representación del poliedro derivado, po

The first of these is the fact that the  
County of York, as it is now, was  
created by the Act of 1844, which  
divided the County of York into  
three parts, the City of York, the  
County of York, and the County of  
West Yorkshire. The County of York  
was then divided into three parts, the  
County of York, the County of  
West Yorkshire, and the County of  
East Yorkshire.

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West Yorkshire.



demostrar observar que éste es un poliedro cóncavo, de

$$C = 3 \times 8 = 24 \text{ caras} \quad (\text{ver lám. 25, fórm. [1]}); \text{ de}$$

$$V = 6 + 8 = 14 \text{ vértices} \quad (\text{ver lám. 25, fórm. [2]}); \text{ y de}$$

$$A = 12 + 3 \times 8 = 36 \text{ aristas} \quad (\text{ver lám. 25, fórm. [3]})$$

La demostración de la concavidad de este poliedro la haremos analíticamente.

### PROCESO GRÁFICO-ANALÍTICO

Calculemos previamente los siguientes valores deducidos de ejercicios anteriores, en función del radio  $a_8$  (dato) de la esfera circunscrita al octaedro regular dado.

Número de caras "n" del octaedro dado

$$n = 8$$

Radio " $a_8$ " de la esfera circunscrita al mismo (dato del ejercicio).

Lado " $l_8$ " del octaedro dado

Se deduce de la fórmula 21, lám. 3

$$l_8 = \frac{2}{\sqrt{2}} a_8 = \sqrt{2} a_8$$

Radio " $b_8$ " de la esfera tangente a las aristas del po-

I hereby certify that the sum of Rs. 100/- (One Hundred Rupees) has been received by Mr. A. B. C. (Name of the person) from Mr. D. E. F. (Name of the person) on the 1st day of Jan. 1920.



This receipt is valid for the purpose of the Government of India (Name of the authority) and is subject to the conditions of the Act of 1919 (Name of the act).

Witness my hand and seal this 1st day of Jan. 1920.

(Signature)

Witness my hand and seal this 1st day of Jan. 1920.

(Signature)

(Signature)

(Signature)

I hereby certify that the sum of Rs. 100/- (One Hundred Rupees) has been received by Mr. A. B. C. (Name of the person) from Mr. D. E. F. (Name of the person) on the 1st day of Jan. 1920.

Diebro regular dado.

Se deduce de la fórmula 22, lám. 3

$$b_8 = b_8 = \frac{1}{2} l_8 = \frac{1}{2} \times \sqrt{2} a_8 = \frac{\sqrt{2}}{2} a_8$$

Radio "c<sub>8</sub>" de la esfera inscrita en el mismo

Se deduce de la fórmula 23, lám. 3.

$$c_8 = \frac{\sqrt{6}}{6} l_8 = \frac{\sqrt{6}}{6} \times \sqrt{2} a_8 = \frac{\sqrt{12}}{6} a_8 = \frac{\sqrt{3}}{3} a_8$$

Radio "d<sub>8</sub>" de la circunferencia circunscrita al polígono regular de una cara del mismo.

Se deduce de la fórmula 24, lám. 3.

$$d_8 = \frac{\sqrt{3}}{3} l_8 = \frac{\sqrt{3}}{3} \times \sqrt{2} a_8 = \frac{\sqrt{6}}{3} a_8$$

Radio "k<sub>8</sub>" de la circunferencia inscrita al polígono regular de una cara del mismo (apotema)

Se deduce de la fórmula 26, lám. 3.

$$k_8 = \frac{\sqrt{3}}{6} l_8 = \frac{\sqrt{3}}{6} \sqrt{2} a_8 = \frac{\sqrt{6}}{6} a_8$$

Ángulo rectilíneo "2 φ<sub>8</sub>" del diedro del mismo

Se deduce de la fórmula 25, lám. 3

$$\text{sen } \varphi_8 = \frac{\sqrt{6}}{3}$$

$$2 \varphi_8 = 109^\circ 28' 16,6''$$

Blank main body area with faint horizontal lines.

Tomando como base los valores anteriores, deduciremos los siguientes del poliedro derivado.

Ángulo rectilíneo "2  $\alpha_8$ " del diedro formado por dos caras contiguas del poliedro derivado, en una arista del octaedro dado.

Se deduce de la fórmula general [4] (ver lám. 25), sustituyendo en ella los valores particulares de este caso.

$$\begin{aligned} \text{tg. } \alpha_8 &= \frac{a_8 k_8}{(k_8)^2 - a_8 c_8 + (c_8)^2} = \frac{a_8 \times \frac{\sqrt{6}}{6} a_8}{\left(\frac{\sqrt{6}}{6} a_8\right)^2 - a_8 \times \frac{\sqrt{3}}{3} a_8 + \left(\frac{\sqrt{3}}{3} a_8\right)^2} = \\ &= -(\sqrt{6} + 2\sqrt{2}) \end{aligned}$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } \boxed{\text{tg. } \alpha_8} &= \frac{a_8 \times \frac{\sqrt{6}}{6} a_8}{\left(\frac{\sqrt{6}}{6} a_8\right)^2 - a_8 \times \frac{\sqrt{3}}{3} a_8 + \left(\frac{\sqrt{3}}{3} a_8\right)^2} = \\ &= \frac{\frac{\sqrt{6}}{6}}{\frac{6}{36} - \frac{\sqrt{3}}{3} + \frac{3}{9}} = \frac{\frac{\sqrt{6}}{6}}{\frac{1}{6} - \frac{\sqrt{3}}{3} + \frac{1}{3}} = \frac{\frac{\sqrt{6}}{6}}{\frac{1 - 2\sqrt{3} + 2}{6}} = \frac{\sqrt{6}}{3 - 2\sqrt{3}} = \\ &= \frac{\sqrt{6}(3 + 2\sqrt{3})}{9 - 12} = -\frac{3\sqrt{6} + 2\sqrt{18}}{3} = -\frac{3\sqrt{6} + 6\sqrt{2}}{3} = \boxed{-(\sqrt{6} + 2\sqrt{2})} \end{aligned}$$

El valor numérico de  $\alpha_8$ , en grados sexagesimales, será:

$$\begin{aligned} \text{tg. } \alpha_8 &= -(\sqrt{6} + 2\sqrt{2}) = -5,2779169\dots = \text{tg. } (\pi - \delta) = -\text{tg. } \delta \\ \text{tg. } \delta &= 5,2779169 \end{aligned}$$





$$\lg \frac{1}{2} \delta = \lg 5, 27 79 16 9.. = 0, 7224 626 \quad \delta = 79^{\circ} 16' 17,1''$$

$$\alpha_8 = 180^{\circ} - 79^{\circ} 16' 17,1'' = 100^{\circ} 43' 42,9'' \quad \gamma \quad 2\alpha_8 = 201^{\circ} 27' 25,8''$$

El valor de  $\alpha_8 > 90^{\circ}$  nos demuestra la concavidad del poliedro derivado (ver lám. 25, "Consideraciones previas").

Altura "p" de una cara lateral de la pirámide recta formada en cada cara del octaedro dado (cara del poliedro derivado).

Se deduce de la fórmula general [5] (ver lám. 25), sustituyendo en ella los valores particulares de este caso.

$$p = \sqrt{(a_8 - c_8)^2 + (k_8)^2} = \sqrt{\left(a_8 - \frac{\sqrt{3}}{3} a_8\right)^2 + \left(\frac{\sqrt{6}}{6} a_8\right)^2} = \sqrt{\frac{9-4\sqrt{3}}{6}} a_8$$

$$\text{Desarrollo del cálculo anterior: } [p] = \sqrt{\left(a_8 - \frac{\sqrt{3}}{3} a_8\right)^2 + \left(\frac{\sqrt{6}}{6} a_8\right)^2} =$$

$$= \sqrt{\left(1 - \frac{\sqrt{3}}{3}\right)^2 a_8^2 + \frac{6}{36} (a_8)^2} = \sqrt{\left(\frac{3-\sqrt{3}}{3}\right)^2 + \frac{1}{6}} \cdot a_8 = \sqrt{\frac{9+3-6\sqrt{3}}{9} + \frac{1}{6}} \cdot a_8 =$$

$$= \sqrt{\frac{12-6\sqrt{3}}{9} + \frac{1}{6}} \cdot a_8 = \sqrt{\frac{4-2\sqrt{3}}{3} + \frac{1}{6}} \cdot a_8 = \sqrt{\frac{8-4\sqrt{3}+1}{6}} a_8 = \boxed{\sqrt{\frac{9-4\sqrt{3}}{6}} a_8}$$

Arista lateral "q" de la pirámide recta regular, o lado igual del triángulo isósceles de una cara del poliedro derivado.

The first part of the document discusses the importance of maintaining accurate records. It states that without proper documentation, it is difficult to track progress and identify areas for improvement. The text emphasizes the need for consistency and thoroughness in record-keeping.

In the second section, the author describes the various methods used to collect and analyze data. This includes both qualitative and quantitative approaches, as well as the use of statistical tools to interpret the results. The importance of choosing the right method for the specific research question is highlighted.

The third part of the document focuses on the challenges faced during the research process. It discusses issues such as limited resources, time constraints, and the potential for bias. The author offers practical advice on how to overcome these challenges and maintain the integrity of the study.

Finally, the document concludes with a summary of the findings and a discussion of their implications. It suggests that the research has provided valuable insights into the topic at hand and offers suggestions for further study in this area.

Se deduce de la fórmula general [6] (ver lám. 25),  
sustituyendo en ella los valores particulares de este caso.

$$q = \sqrt{(a_8 - c_8)^2 + (d_8)^2} = \sqrt{\left(a_8 - \frac{\sqrt{3}}{3} a_8\right)^2 + \left(\frac{\sqrt{6}}{3} a_8\right)^2} = \sqrt{\frac{6 - 2\sqrt{3}}{3}} a_8$$

Desarrollo del cálculo anterior:  $\boxed{q} = \sqrt{\left(a_8 - \frac{\sqrt{3}}{3} a_8\right)^2 + \left(\frac{\sqrt{6}}{3} a_8\right)^2} =$

$$= \sqrt{\left(1 - \frac{\sqrt{3}}{3}\right)^2 + \frac{6}{9}} \cdot a_8 = \sqrt{\left(\frac{3 - \sqrt{3}}{3}\right)^2 + \frac{6}{9}} \cdot a_8 = \sqrt{\frac{9 + 3 - 6\sqrt{3}}{9} + \frac{6}{9}} \cdot a_8 =$$

$$= \sqrt{\frac{18 - 6\sqrt{3}}{9}} a_8 = \boxed{\sqrt{\frac{6 - 2\sqrt{3}}{3}} a_8}$$

Segmento "t" que se obtiene al unir los extremos  
de dos lados consecutivos del polígono de una cara  
del octaedro dado.

Es el tercer lado del triángulo equilátero de  
la cara, por lo que

$$t = t_8 = \sqrt{2} \cdot a_8$$

Ángulo rectilíneo del diedro "28" formado por dos  
caras laterales contiguas en las aristas de la pirámide recta.

Se deduce de la fórmula general [7] (ver lám. 25),  
sustituyendo en ella los valores particulares de este caso.

$$\text{sen } \gamma_8 = \frac{t q}{2 t_8 p} = \frac{\sqrt{2} a_8 \sqrt{\frac{6 - 2\sqrt{3}}{3}} \cdot a_8}{2 \times \sqrt{2} a_8 \cdot \sqrt{\frac{9 - 4\sqrt{3}}{6}} a_8} = \sqrt{\frac{5 + \sqrt{3}}{11}}$$

$$f(x) = \frac{1}{2} \left( \sqrt{x^2 + 4} + \sqrt{x^2 - 4} \right) \quad (1)$$

$$f(x) = \frac{1}{2} \left( \sqrt{x^2 + 4} + \sqrt{x^2 - 4} \right) \quad (2)$$

$$f(x) = \frac{1}{2} \left( \sqrt{x^2 + 4} + \sqrt{x^2 - 4} \right) \quad (3)$$

$$f(x) = \frac{1}{2} \left( \sqrt{x^2 + 4} + \sqrt{x^2 - 4} \right) \quad (4)$$

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

The second part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{2} \left( \sqrt{x^2 + 4} + \sqrt{x^2 - 4} \right) \quad (5)$$

$$f(x) = \frac{1}{2} \left( \sqrt{x^2 + 4} + \sqrt{x^2 - 4} \right) \quad (6)$$

The third part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{2} \left( \sqrt{x^2 + 4} + \sqrt{x^2 - 4} \right) \quad (7)$$



Desarrollo del cálculo anterior:  $\boxed{\text{sen } \gamma_8} = \frac{\sqrt{2} a_8 \sqrt{\frac{6-2\sqrt{3}}{3}} a_8}{2 \sqrt{2} a_8 \sqrt{\frac{9-4\sqrt{3}}{6}} a_8} =$

$$= \frac{\sqrt{\frac{6-2\sqrt{3}}{3}}}{2 \sqrt{\frac{9-4\sqrt{3}}{6}}} = \frac{1}{2} \cdot \sqrt{\frac{6-2\sqrt{3}}{3} \cdot \frac{9-4\sqrt{3}}{6}} = \frac{1}{2} \sqrt{\frac{2(6-2\sqrt{3})}{9-4\sqrt{3}}} =$$

$$= \frac{1}{2} \sqrt{\frac{2(6-2\sqrt{3})(9+4\sqrt{3})}{81-48}} = \frac{1}{2} \sqrt{\frac{2(54-12\sqrt{3}+24\sqrt{3}-24)}{33}} =$$

$$= \frac{1}{2} \sqrt{\frac{2(30+6\sqrt{3})}{33}} = \frac{1}{2} \sqrt{\frac{2(10+2\sqrt{3})}{11}} = \frac{1}{2} \sqrt{\frac{4(5+\sqrt{3})}{11}} = \boxed{\sqrt{\frac{5+\sqrt{3}}{11}}}$$

$$\text{sen } \gamma_8 = \sqrt{\frac{5+\sqrt{3}}{11}} = 0,78 \ 23 \ 07 \ 2 \quad \text{lg sen } \gamma_8 = \bar{1},8933 \ 773$$

$$\gamma_8 = 51^\circ \ 28' \ 20,3''$$

$$2 \gamma_8 = 102^\circ \ 56' \ 40,6''$$

Ángulo diedro " $\beta_8$ " formado por una cara lateral de la pirámide y en base

Se deduce de la fórmula general [8] (ver condiciones previas, lám. 25), substituyendo en ella los valores particulares de este caso.

$$\text{sen } \beta_8 = \frac{a_8 - c_8}{p} = \frac{a_8 - \frac{\sqrt{3}}{3} a_8}{\sqrt{\frac{9-4\sqrt{3}}{6}} a_8} = \sqrt{\frac{24-4\sqrt{3}}{33}}$$

Desarrollo del cálculo anterior:  $\boxed{\text{sen } \beta_8} = \frac{a_8 - \frac{\sqrt{3}}{3} a_8}{\sqrt{\frac{9-4\sqrt{3}}{6}} a_8} =$

Q.1. Find the value of  $\sin^{-1}(\sin \frac{\pi}{6})$

Sol. We know that  $\sin^{-1}(\sin x) = x$  if  $x$  lies in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .

Here,  $\frac{\pi}{6}$  lies in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .

$\therefore \sin^{-1}(\sin \frac{\pi}{6}) = \frac{\pi}{6}$

Q.2. Find the value of  $\cos^{-1}(\cos \frac{5\pi}{6})$

Sol. We know that  $\cos^{-1}(\cos x) = x$  if  $x$  lies in  $[0, \pi]$ .

Here,  $\frac{5\pi}{6}$  lies in  $[0, \pi]$ .

$\therefore \cos^{-1}(\cos \frac{5\pi}{6}) = \frac{5\pi}{6}$

Q.3. Find the value of  $\tan^{-1}(\tan \frac{2\pi}{3})$

$$\begin{aligned}
 &= \frac{1 - \frac{\sqrt{3}}{3}}{\sqrt{\frac{9 - 4\sqrt{3}}{6}}} = \frac{\frac{3 - \sqrt{3}}{3} \sqrt{\frac{9 - 4\sqrt{3}}{6}}}{\frac{9 - 4\sqrt{3}}{6}} = \left( \frac{3 - \sqrt{3}}{3} : \frac{9 - 4\sqrt{3}}{6} \right) \times \sqrt{\frac{9 - 4\sqrt{3}}{6}} = \\
 &= \frac{6 \cdot (3 - \sqrt{3})}{3 \cdot (9 - 4\sqrt{3})} \times \sqrt{\frac{9 - 4\sqrt{3}}{6}} = \frac{2(3 - \sqrt{3})(9 + 4\sqrt{3})}{81 - 48} \cdot \sqrt{\frac{9 - 4\sqrt{3}}{6}} = \\
 &= \frac{2(27 - 9\sqrt{3} + 12\sqrt{3} - 12)}{33} \sqrt{\frac{9 - 4\sqrt{3}}{6}} = \frac{2 \cdot (15 + 3\sqrt{3})}{33} \sqrt{\frac{9 - 4\sqrt{3}}{6}} = \\
 &= \frac{2(5 + \sqrt{3})}{11} \sqrt{\frac{9 - 4\sqrt{3}}{6}} = \frac{2}{11} \sqrt{\frac{(9 - 4\sqrt{3})(5 + \sqrt{3})^2}{6}} = \frac{2}{11} \sqrt{\frac{(9 - 4\sqrt{3})(25 + 3 + 10\sqrt{3})}{6}} = \\
 &= \frac{2}{11} \sqrt{\frac{(9 - 4\sqrt{3})(28 + 10\sqrt{3})}{6}} = \frac{2}{11} \sqrt{\frac{(9 - 4\sqrt{3})(14 + 5\sqrt{3})}{3}} = \frac{2}{11} \sqrt{\frac{126 - 56\sqrt{3} + 45\sqrt{3} - 60}{3}} = \\
 &= \frac{2}{11} \sqrt{\frac{66 - 11\sqrt{3}}{3}} = 2 \sqrt{\frac{11(6 - \sqrt{3})}{3 \times 11^2}} = \sqrt{\frac{4(6 - \sqrt{3})}{33}} = \boxed{\sqrt{\frac{24 - 4\sqrt{3}}{33}}}
 \end{aligned}$$

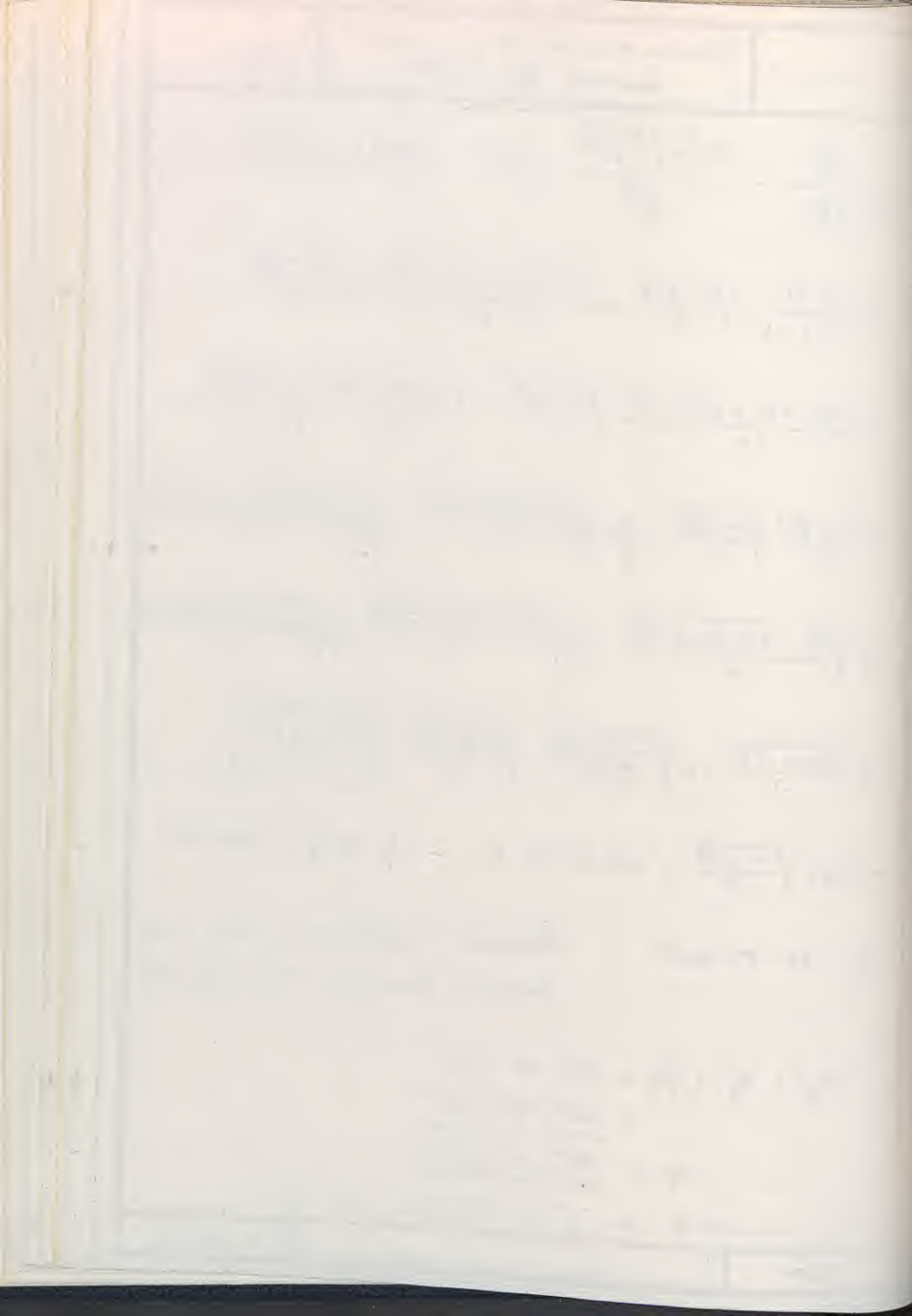
$$\text{sen } \beta_8 = \sqrt{\frac{24 - 4\sqrt{3}}{33}} = 0,7192546... \quad \text{tg sen } \beta_8 = 7,8568827$$

$$\beta_8 = 45^\circ 59' 34,6''$$

debiendo verificarse como comprobación (ver fórm. [11], lám. 25)

$$\begin{aligned}
 \alpha_8 &= \varphi_8 + \beta_8 = 54^\circ 44' 8,3'' \\
 &\quad + 45^\circ 59' 34,6'' \\
 \alpha_8 &= \underline{\underline{100^\circ 43' 42,9''}}
 \end{aligned}$$

valor coincidente con el ya obtenido de  $\alpha_8$





Radio " $b_2$ " de la esfera tangente a las aristas laterales de las pirámides rectas cuyas bases son caras del octaedro regular dado.

Se deduce de la fórmula general [9] (ver consideraciones previas, lám. 25), sustituyendo en ella los valores particulares de este caso

$$b_2 = \sqrt{(a_2)^2 - \frac{9^2}{4}} = \sqrt{(a_2)^2 - \left(\sqrt{\frac{6-2\sqrt{3}}{3}} a_2\right)^2 : 4} = \sqrt{\frac{3+\sqrt{3}}{6}} a_2$$

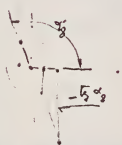
$$\begin{aligned} \text{Desarrollo del cálculo anterior: } \boxed{b_2} &= \sqrt{(a_2)^2 - \left(\sqrt{\frac{6-2\sqrt{3}}{3}} a_2\right)^2 : 4} = \\ &= \sqrt{1 - \frac{6-2\sqrt{3}}{3 \times 4}} \cdot a_2 = \sqrt{1 - \frac{3-\sqrt{3}}{6}} a_2 = \sqrt{\frac{6-3+\sqrt{3}}{6}} a_2 = \boxed{\sqrt{\frac{3+\sqrt{3}}{6}} a_2} \end{aligned}$$

Radio " $c_1$ " de la esfera inscrita en el poliedro derivado.

Se deduce de la fórmula general [10] (ver lám. 25) sustituyendo en ella los valores particulares de este caso.

$$c_1 = b_1 \operatorname{sen} \alpha_2, \quad \text{siendo } b_1 = \frac{\sqrt{2}}{2} a_2 \quad \text{y } \tan \alpha_2 = -(\sqrt{6} + 2\sqrt{2})$$

$$\text{pero } \operatorname{sen} \alpha_2 = \frac{\tan \alpha_2}{\sqrt{1 + \tan^2 \alpha_2}} = \frac{-(\sqrt{6} + 2\sqrt{2})}{\sqrt{1 + (-\sqrt{6} + 2\sqrt{2})^2}} = -\sqrt{\frac{18 + 8\sqrt{3}}{33}}$$



pero estando  $\alpha_2$  comprendido entre  $\pi$  y  $\frac{3\pi}{2}$ , el  $\operatorname{sen} \alpha_2$  será positivo, por lo que

$$\operatorname{sen} \alpha_2 = \sqrt{\frac{18 + 8\sqrt{3}}{33}}$$



The first part of the paper is devoted to a study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \int_0^x \frac{1}{1+t^2} dt$$

It is well known that this function is increasing and concave down on the interval  $(-\infty, \infty)$ . We shall now prove that it is also bounded.

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \int_0^x \frac{1}{1+t^2} dt = \int_0^\infty \frac{1}{1+t^2} dt$$

This integral converges, and its value is  $\frac{\pi}{2}$ .

Similarly, we can show that  $\lim_{x \rightarrow -\infty} f(x) = -\frac{\pi}{2}$ . Therefore, the function  $f(x)$  is bounded on the entire real line.

$$f(x) = \frac{\pi}{2} - \int_x^\infty \frac{1}{1+t^2} dt$$

This representation of the function  $f(x)$  is useful in many applications. For example, it can be used to estimate the error in the approximation of  $f(x)$  by a finite sum.

Desarrollo del cálculo anterior:  $\boxed{\cos \alpha_8} = \frac{-(\sqrt{6} + 2\sqrt{2})}{\sqrt{1 + (-(\sqrt{6} + 2\sqrt{2}))^2}} =$

$$= - \frac{\sqrt{6} + 2\sqrt{2}}{\sqrt{1 + (\sqrt{6} + 2\sqrt{2})^2}} = - \frac{\sqrt{6} + 2\sqrt{2}}{\sqrt{1 + 6 + 8 + 4\sqrt{12}}} = - \frac{\sqrt{6} + 2\sqrt{2}}{\sqrt{15 + 8\sqrt{3}}} =$$

$$= - \frac{(\sqrt{6} + 2\sqrt{2}) \times \sqrt{15 + 8\sqrt{3}}}{15 + 8\sqrt{3}} = - \frac{(\sqrt{6} + 2\sqrt{2})(15 - 8\sqrt{3})}{15^2 - 64 \times 3} \times \sqrt{15 + 8\sqrt{3}} =$$

$$= - \frac{15\sqrt{6} + 30\sqrt{2} - 8\sqrt{18} - 16\sqrt{6}}{33} \times \sqrt{15 + 8\sqrt{3}} = - \frac{6\sqrt{2} - \sqrt{6}}{3} \times \sqrt{15 + 8\sqrt{3}} =$$

$$= - \frac{1}{33} \times \sqrt{(15 + 8\sqrt{3})(6\sqrt{2} - \sqrt{6})^2} = - \frac{1}{33} \sqrt{(15 + 8\sqrt{3})(72 + 6 - 12\sqrt{12})} =$$

$$= - \frac{1}{33} \times \sqrt{(15 + 8\sqrt{3})(78 - 24\sqrt{3})} = - \frac{1}{33} \sqrt{6 \times (15 + 8\sqrt{3})(13 - 4\sqrt{3})} =$$

$$= - \frac{1}{33} \sqrt{6 \times (195 + 104\sqrt{3} - 60\sqrt{3} - 96)} = - \frac{1}{33} \sqrt{6 \times (99 + 44\sqrt{3})} =$$

$$= - \frac{1}{33} \sqrt{6 \times 11 \times (9 + 4\sqrt{3})} = - \sqrt{\frac{6 \times 11 \times (9 + 4\sqrt{3})}{33 \times 33}} = - \sqrt{\frac{2 \times (9 + 4\sqrt{3})}{33}} =$$

$$= - \sqrt{\frac{18 + 8\sqrt{3}}{33}}$$

y por consiguiente, tendremos pues

$$c_1 = b_1 \quad \cos \alpha_8 = \frac{\sqrt{2}}{2} \times a_8 \times \sqrt{\frac{18 + 8\sqrt{3}}{33}} = \sqrt{\frac{9 + 4\sqrt{3}}{33}} a_8$$

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{x} \int_0^x t f(t) dt$$

for  $x > 0$ . It is shown that the function  $f(x)$  is continuous and differentiable for  $x > 0$  and that it satisfies the differential equation

$$x f'(x) + f(x) = 0$$

for  $x > 0$ . The general solution of this equation is

$$f(x) = \frac{C}{x}$$

where  $C$  is an arbitrary constant. It is also shown that the function  $f(x)$  is bounded for  $x > 0$  if and only if  $C = 0$ .

In the second part of the paper, we consider the problem of finding the maximum and minimum values of the function  $f(x)$  on the interval  $[a, b]$ . It is shown that the function  $f(x)$  has a local maximum at  $x = a$  and a local minimum at  $x = b$  if and only if  $C > 0$  and  $C < 0$  respectively.

Finally, we consider the problem of finding the values of  $x$  for which the function  $f(x)$  is increasing or decreasing. It is shown that the function  $f(x)$  is increasing for  $x > 0$  if and only if  $C < 0$  and decreasing for  $x > 0$  if and only if  $C > 0$ .

Este mismo valor se puede deducir de la fórmula equivalente  $[10']$  (lámin. 25), en la que

$$c_1 = b_2 \text{ sea } \gamma_8 = \sqrt{\frac{3 + \sqrt{3}}{6}} a_8 \sqrt{\frac{5 + \sqrt{3}}{11}} = \sqrt{\frac{9 + 4\sqrt{3}}{33}} a_8$$

Desarrollo del cálculo anterior:  $[C_1] = \sqrt{\frac{3 + \sqrt{3}}{6}} a_8 \sqrt{\frac{5 + \sqrt{3}}{11}} =$

$$= \sqrt{\frac{(3 + \sqrt{3})(5 + \sqrt{3})}{6 \times 11}} a_8 = \sqrt{\frac{15 + 5\sqrt{3} + 3\sqrt{3} + 3}{6 \times 11}} a_8 = \sqrt{\frac{18 + 8\sqrt{3}}{6 \times 11}} a_8 =$$

$$= \sqrt{\frac{9 + 4\sqrt{3}}{33}} a_8$$

### Área lateral "S" del poliedro derivado

Se obtiene como suma de las áreas laterales de las ocho pirámides rectas de base triangular <sup>(de lado " $l_8$ ")</sup> equilátera, cuyas caras laterales son triángulos isósceles de base " $l_8$ " y altura " $p$ ", ya determinados.

$$S = 8 \times 3 \times \frac{l_8 \cdot p}{2} = 12 \times \sqrt{2} a_8 \times \sqrt{\frac{9 - 4\sqrt{3}}{6}} a_8 = 12 \sqrt{\frac{9 - 4\sqrt{3}}{3}} (a_8)^2$$

### Volumen "V" del poliedro derivado

Se obtiene como suma del volumen del octaedro dado y de las ocho pirámides formadas en sus caras.

$$V = V_8 + 8 \times \frac{S_3 \times h}{3}$$

Consider the function  $f(x) = x^2 + 2x + 1$ . The derivative of  $f(x)$  is  $f'(x) = 2x + 2$ .

$$f'(x) = 2x + 2 = 2(x + 1)$$

Setting  $f'(x) = 0$ , we find the critical points where the function has a local maximum or minimum.

$$2(x + 1) = 0 \implies x + 1 = 0 \implies x = -1$$

At  $x = -1$ , the function has a local maximum. The value of the function at this point is  $f(-1) = 0$ .

Since the function is a parabola opening upwards, the local maximum at  $x = -1$  is also the global maximum.

The function  $f(x) = x^2 + 2x + 1$  can be written as  $f(x) = (x + 1)^2$ . This shows that the function is always non-negative.

The minimum value of the function is  $0$ , which occurs at  $x = -1$ .

$$f(x) = (x + 1)^2 \geq 0$$

The function  $f(x) = (x + 1)^2$  is symmetric about the line  $x = -1$ .

The graph of the function is a parabola opening upwards with its vertex at  $(-1, 0)$ .

The function  $f(x) = (x + 1)^2$  is increasing for  $x > -1$  and decreasing for  $x < -1$ .



siendo " $S_3$ " el área de una cara del octaedro, y " $h$ " la altura de la pirámide.

Para obtener  $V_8$  en función de  $a_8$ , ver lám. 3, fórmulas 28 y 21, que nos dan

$$V_8 = \frac{\sqrt{2}}{3} (l_8)^3 = \frac{\sqrt{2}}{3} \times (\sqrt{2} a_8)^3 = \frac{4}{3} (a_8)^3$$

Por otra parte, tendremos que

$$S_3 = \frac{\sqrt{3}}{4} (l_8)^2 = \frac{\sqrt{3}}{4} \times (\sqrt{2} a_8)^2 = \frac{\sqrt{3}}{2} (a_8)^2$$

y también que (ver lám. 3, fórmulas 21 y 23)

$$h = a_8 - c_8 = a_8 - \frac{\sqrt{6}}{6} l_8 = a_8 - \frac{\sqrt{6}}{6} \times \sqrt{2} a_8 = \frac{\sqrt{12}}{6} a_8 = \frac{\sqrt{3}}{3} a_8$$

y finalmente

$$V = V_8 + 8 \times \frac{S_3 \times h}{3} = \frac{4}{3} (a_8)^3 + \frac{8}{3} \times \frac{\sqrt{3}}{2} (a_8)^2 \times \frac{\sqrt{3}}{3} a_8 = \frac{8}{3} (a_8)^3$$

$$\text{Desarrollo del cálculo anterior: } V = \frac{4}{3} (a_8)^3 + \frac{8}{3} \times \frac{\sqrt{3}}{2} (a_8)^2 \times \frac{\sqrt{3}}{3} a_8 =$$

$$= \left( \frac{4}{3} + \frac{8}{3} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{3} \right) (a_8)^3 = \left( \frac{4}{3} + \frac{4}{3} \right) (a_8)^3 = \frac{8}{3} (a_8)^3$$

Cuyo resultado nos demuestra que "El volumen del poliedro derivado del octaedro regular es el doble del volumen de éste"

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined on the interval  $[0, 1]$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \frac{1}{x^2}$ . This result is used to prove that  $f(x)$  is a convex function on  $[0, 1]$ .

In the second part, we consider the function  $g(x) = \ln x$  on the interval  $(0, 1]$ . It is shown that  $g(x)$  is concave on this interval. This is done by showing that the second derivative of  $g(x)$  is negative. The function  $g(x)$  is also shown to be increasing on  $(0, 1]$ .

The third part of the paper deals with the function  $h(x) = x \ln x$  on the interval  $(0, 1]$ . It is shown that  $h(x)$  has a maximum at  $x = \frac{1}{e}$ . This is done by finding the critical points of  $h(x)$  and showing that  $x = \frac{1}{e}$  is the only one in the interval  $(0, 1]$ .

Finally, we consider the function  $k(x) = x^2 \ln x$  on the interval  $(0, 1]$ . It is shown that  $k(x)$  has a minimum at  $x = \frac{1}{e}$ . This is done by finding the critical points of  $k(x)$  and showing that  $x = \frac{1}{e}$  is the only one in the interval  $(0, 1]$ .

En el cuadro sinóptico que damos a continuación,  
 resumimos los resultados anteriores:

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
<sup>268</sup> $l_8$	$\sqrt{2} \ a_8$	1, 41 42 14... $a_8$
<sup>269</sup> $b_1$	$\frac{\sqrt{2}}{2} \ a_8$	0, 70 71 07... $a_8$
<sup>270</sup> $b_2$	$\sqrt{\frac{3+\sqrt{3}}{6}} \ a_8$	0, 88 80 74... $a_8$
<sup>271</sup> $c_8$	$\frac{\sqrt{3}}{3} \ a_8$	0, 57 73 50... $a_8$
$c_1$	$\sqrt{\frac{9+4\sqrt{3}}{33}} \ a_8$	0, 69 47 47... $a_8$
$d_8$	$\frac{\sqrt{6}}{3} \ a_8$	0, 81 64 97... $a_8$
$k_8$	$\frac{\sqrt{6}}{6} \ a_8$	0, 40 82 48... $a_8$
$2 \varphi_8$	$\text{sen } \varphi_8 = \frac{\sqrt{6}}{3}$	$\text{sen } \varphi_8 = 0, 81 \ 64 \ 97$ $2 \varphi_8 = 109^\circ \ 28' \ 16,6''$
$2 \alpha_8$	$\text{tg } 2 \alpha_8 = -(\sqrt{6} + 2\sqrt{2})$	$\text{tg } \alpha_8 = -5, 27 \ 79 \ 17$ $2 \alpha_8 = 207^\circ \ 27' \ 25,8''$
$2 \gamma_8$	$\text{sen } \gamma_8 = \sqrt{\frac{5+\sqrt{3}}{11}}$	$\text{sen } \gamma_8 = 0, 78 \ 23 \ 07$ $2 \gamma_8 = 102^\circ \ 56' \ 40,6''$
$\beta_8$	$\text{sen } \beta_8 = \sqrt{\frac{24-4\sqrt{3}}{33}}$	$\text{sen } \beta_8 = 0, 71 \ 92 \ 55$ $\beta_8 = 45^\circ \ 59' \ 34,6''$
$p$	$\sqrt{\frac{9-4\sqrt{3}}{6}} \ a_8$	0, 58 76 22... $a_8$
$q$	$\sqrt{\frac{6-2\sqrt{3}}{3}} \ a_8$	0, 91 94 02... $a_8$
$t$	$\sqrt{2} \ a_8$	1, 41 42 14... $a_8$
$S$	$12 \sqrt{\frac{9-4\sqrt{3}}{3}} \ (a_8)^2$	9, 97 22 74... $(a_8)^2$
$V$	$\frac{8}{3} \ (a_8)^3$	2, 66 66 67... $(a_8)^3$

Subject: *Mathematics*

Chapter: *Algebra*

Topic: *Quadratic Equations*

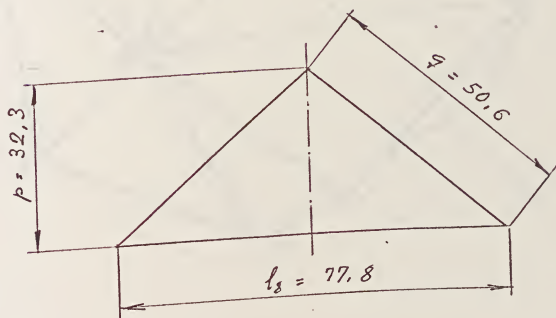
### Exercise 1.1

Q. No.	Answer	Q. No.	Answer
1. Find the roots of the equation $x^2 - 5x + 6 = 0$ .	$x = 2, 3$	11. Find the roots of the equation $x^2 - 12x + 36 = 0$ .	$x = 6$
2. Find the roots of the equation $x^2 + 7x + 12 = 0$ .	$x = -3, -4$	12. Find the roots of the equation $x^2 - 15x + 54 = 0$ .	$x = 6, 9$
3. Find the roots of the equation $x^2 - 8x + 15 = 0$ .	$x = 3, 5$	13. Find the roots of the equation $x^2 + 9x + 14 = 0$ .	$x = -2, -7$
4. Find the roots of the equation $x^2 - 10x + 21 = 0$ .	$x = 3, 7$	14. Find the roots of the equation $x^2 - 11x + 28 = 0$ .	$x = 4, 7$
5. Find the roots of the equation $x^2 + 13x + 42 = 0$ .	$x = -6, -7$	15. Find the roots of the equation $x^2 - 17x + 72 = 0$ .	$x = 8, 9$
6. Find the roots of the equation $x^2 - 19x + 90 = 0$ .	$x = 10, 9$	16. Find the roots of the equation $x^2 + 21x + 110 = 0$ .	$x = -10, -11$
7. Find the roots of the equation $x^2 - 21x + 110 = 0$ .	$x = 10, 11$	17. Find the roots of the equation $x^2 - 23x + 132 = 0$ .	$x = 12, 11$
8. Find the roots of the equation $x^2 + 25x + 150 = 0$ .	$x = -10, -15$	18. Find the roots of the equation $x^2 - 25x + 144 = 0$ .	$x = 16, 9$
9. Find the roots of the equation $x^2 - 27x + 182 = 0$ .	$x = 14, 13$	19. Find the roots of the equation $x^2 + 29x + 210 = 0$ .	$x = -14, -15$
10. Find the roots of the equation $x^2 - 29x + 210 = 0$ .	$x = 14, 15$	20. Find the roots of the equation $x^2 - 31x + 240 = 0$ .	$x = 16, 15$

FIGURA CORPÓREA

Se obtiene por acoplamiento de 24 caras iguales en forma de triángulos isósceles, de base  $l_3 = 77,8$  mm y altura  $p = 32,3$  mm; en este triángulo el lado  $q$ , tiene el valor  $q = 50,6$  mm (comprobación).

Para obtener este poliedro se formarán previamente 2 pirámides rectas de base triangular equilátera de lado " $l_3$ ," cuyas caras laterales son 3 triángulos (ver figura) acoplados por su lado " $q$ "

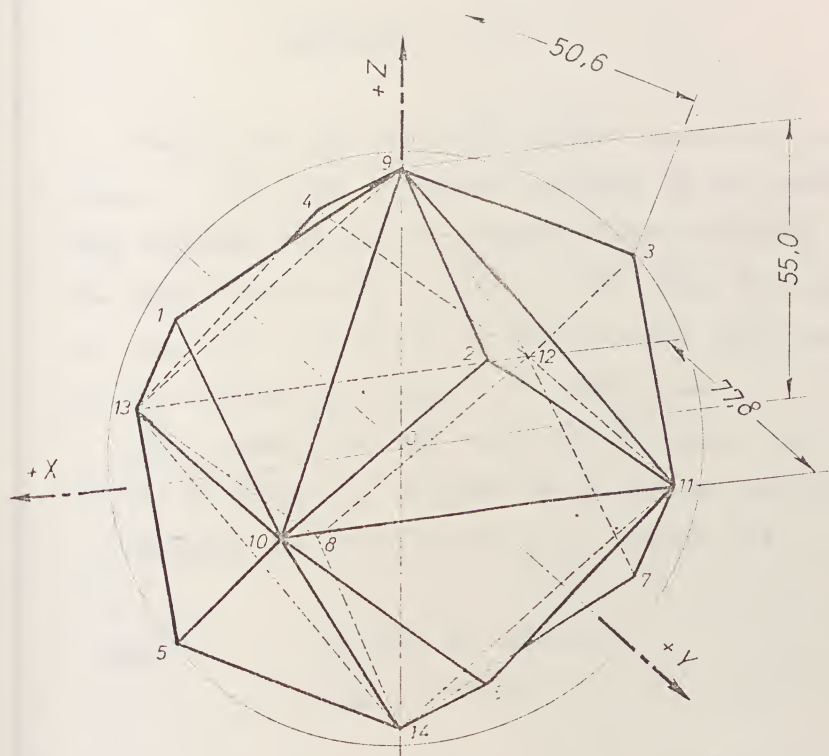




On the theory of the

In this paper we shall consider the problem of the
 construction of the
  $n$ -gonal numbers. The
  $n$ -gonal numbers are defined by the formula
 
$$P_n(x) = \frac{n-1}{2}x^2 - \frac{n-3}{2}x$$
 where  $x$  is a positive integer. The
  $n$ -gonal numbers are called
  $n$ -gonal numbers because they are the
 numbers of points in a regular
  $n$ -gonal arrangement of points.
 The
  $n$ -gonal numbers are also called
  $n$ -gonal numbers because they are the
 numbers of points in a regular
  $n$ -gonal arrangement of points.





*Poliedro derivado del octaedro regular*



Figure 1. A diagram showing the relationship between the variables  $x$  and  $y$ .

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro derivado de un dodecaedro regular, obtenido al proyectar desde el centro de la esfera circunscrita a éste, y sobre ella, los centros de cada cara, uniendo a continuación estos puntos con los vértices del polígono de dicha cara.

Las coordenadas del centro de la esfera, son:  
 $O (72, 72, 85) \text{ mm}$  y el radio de la misma, de 55 mm.

Dibujar en formato A3V y a escala 1:1.

DATOS

$O (72, 72, 85) \text{ mm}$

$\phi_{12} = 55 \text{ mm.}$

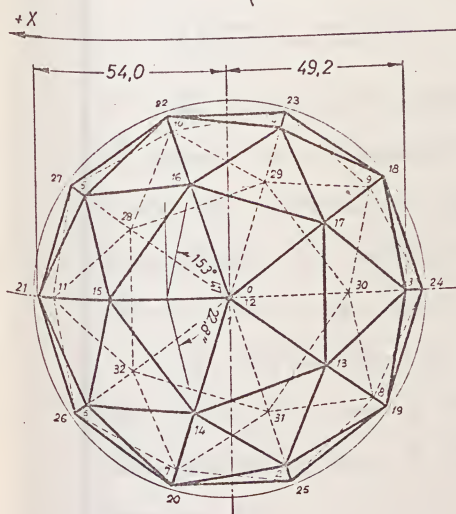
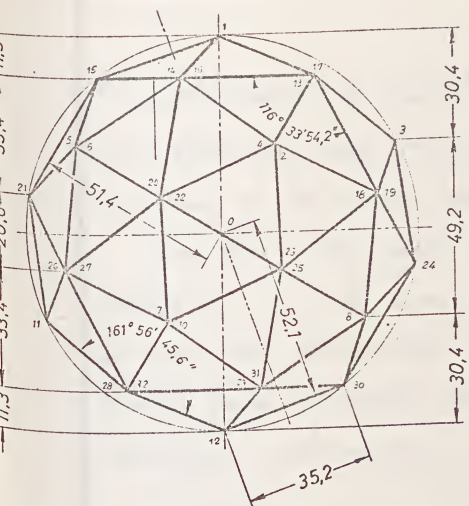
ORIGINAL ARTICLES

1. The Effect of the Diet on the Growth of the Child  
by Dr. J. H. Tappan, New York, N. Y.  
The diet of the child is a factor of great importance in the development of the body and mind. The diet should be such as to supply the body with the necessary foodstuffs for the growth and maintenance of the body. The diet should be such as to supply the body with the necessary foodstuffs for the growth and maintenance of the body. The diet should be such as to supply the body with the necessary foodstuffs for the growth and maintenance of the body.

Published by the American Medical Association  
535 North Dearborn Street, Chicago, Ill.



I

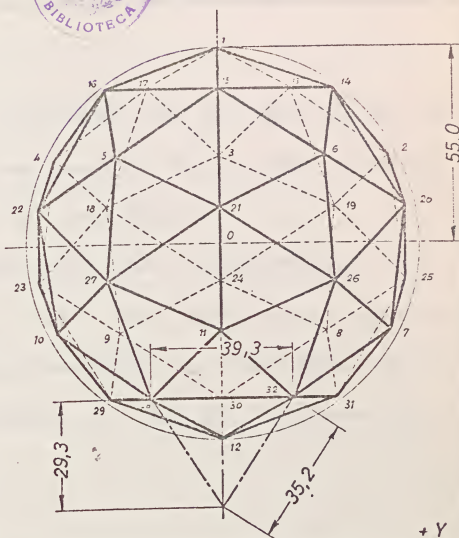


#### NUMERACIÓN DE VÉRTICES

Dodecaedro regular..... 13 al 32  
 Proyecciones centros caras del mismo  
 (vértices del icosaedro conjugado).... 1 al 12



III



#### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro derivado de un dodecaedro regular, obtenido al proyectar desde el centro de la esfera circunscrita a éste, y sobre ella, los centros de cada cara, uniendo a continuación estos puntos con los vértices del polígono de dicha cara.

Las coordenadas del centro de la esfera son:  $O(72, 72, 85)$  mm y el radio de la misma de 55 mm.

Dibujar en formato A3v y a escala 1:1.

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala	Poliedro derivado del dodecaedro regular				Lámina 28
1:1					Curso 19 - 19

II



The first diagram is a circle with a square inscribed within it. The square is rotated 45 degrees, so its vertices touch the midpoints of the circle's sides. Lines connect the corners of the square to the points where the square's sides touch the circle's circumference. These lines intersect to form a smaller square in the center of the circle.



The second diagram is a circle with a square inscribed within it. The square is rotated 45 degrees, so its vertices touch the midpoints of the circle's sides. Lines connect the corners of the square to the points where the square's sides touch the circle's circumference. These lines intersect to form a smaller square in the center of the circle.

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96
97	98	99	100	101	102	103	104	105	106	107	108
109	110	111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130	131	132
133	134	135	136	137	138	139	140	141	142	143	144
145	146	147	148	149	150	151	152	153	154	155	156
157	158	159	160	161	162	163	164	165	166	167	168
169	170	171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190	191	192
193	194	195	196	197	198	199	200	201	202	203	204
205	206	207	208	209	210	211	212	213	214	215	216
217	218	219	220	221	222	223	224	225	226	227	228
229	230	231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250	251	252
253	254	255	256	257	258	259	260	261	262	263	264
265	266	267	268	269	270	271	272	273	274	275	276
277	278	279	280	281	282	283	284	285	286	287	288
289	290	291	292	293	294	295	296	297	298	299	300

Al estudiar el ejercicio propuesto en la lámina 25, hemos obtenido unas deducciones previas de carácter general, comunes a los cinco poliedros regulares.

Las fórmulas allí deducidas las aplicaremos sucesivamente en este caso particular del dodecaedro regular. El desarrollo del cálculo correspondiente a esta lámina, seguirá pues aquellas directrices, a las que haremos las oportunas referencias.

#### PROCESO GRÁFICO

En el caso del poliedro derivado del dodecaedro regular, el proceso es inmediato, ya que sabemos que el conjugado del dodecaedro regular es un icosaedro, y esta representación ha sido ya realizada en el ejercicio de la lámina 24, cuyo proceso nos permite:

1º Representar el dodecaedro regular dado, de vértices 13 al 32, inscrito en una esfera de 55 mm de radio (estos vértices se han de corresponder con los 21 al 40 de la lámina 24).

2º Obtener los vértices 1 al 12 del icosaedro conjugado inscrito en la misma esfera.

3º Unir los vértices 1 al 12 con los correspondientes de cada cara del dodecaedro dado.

The first of these is the fact that the  
the second is the fact that the  
the third is the fact that the  
the fourth is the fact that the  
the fifth is the fact that the

the sixth is the fact that the  
the seventh is the fact that the  
the eighth is the fact that the  
the ninth is the fact that the  
the tenth is the fact that the  
the eleventh is the fact that the  
the twelfth is the fact that the  
the thirteenth is the fact that the  
the fourteenth is the fact that the  
the fifteenth is the fact that the

the sixteenth is the fact that the  
the seventeenth is the fact that the  
the eighteenth is the fact that the  
the nineteenth is the fact that the  
the twentieth is the fact that the



Al terminar la representación del poliedro derivado, podemos comprobar que éste es un poliedro convexo, de

$$\begin{aligned} C &= 5 \times 12 = 60 \text{ caras} && (\text{ver lám. 25, fórm. [1]}); && \text{de} \\ V &= 12 + 20 = 32 \text{ vértices} && (\text{ver lám. 25, fórm. [2]}); && \text{y de} \\ A &= 30 + 5 \times 12 = 90 \text{ aristas} && (\text{ver lám. 25, fórm. [3]}). \end{aligned}$$

La demostración de la convexidad de este poliedro la haremos analíticamente.

#### PROCESO GRÁFICO-ANALÍTICO

Calcularemos previamente los siguientes valores deducidos de ejercicios anteriores, en función del radio  $a_{12}$  (dato) de la esfera circunscrita al dodecaedro regular dado.

Número de caras "n" del dodecaedro dado

$$n = 12$$

Radio " $a_{12}$ " de la esfera circunscrita al mismo (dato del ejercicio).

Lado " $l_{12}$ " del dodecaedro dado.

Se deduce de la fórm. 30, lám. 4

$$l_{12} = \frac{4}{\sqrt{15} + \sqrt{3}} a_{12} = \frac{4(\sqrt{15} - \sqrt{3})}{12} a_{12} = \frac{\sqrt{15} - \sqrt{3}}{3} a_{12}$$



The first part of the document is a letter from the Secretary of the Board of Directors to the Board of Directors. The letter is dated 10th day of January, 1900, and is addressed to the Board of Directors of the Company.

The letter contains a report on the business of the Company during the year 1899. The report is a summary of the business of the Company during the year 1899, and is a summary of the business of the Company during the year 1899.

The second part of the document is a letter from the Secretary of the Board of Directors to the Board of Directors. The letter is dated 10th day of January, 1900, and is addressed to the Board of Directors of the Company.

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The letter contains a report on the business of the Company during the year 1899. The report is a summary of the business of the Company during the year 1899, and is a summary of the business of the Company during the year 1899.

Radio "b<sub>1</sub>" de la esfera tangente a las aristas del poliedro regular dado.

Se deduce de la fórmula 31, lám. 4

$$b_1 = b_{12} = \frac{3 + \sqrt{5}}{4} \cdot \frac{3 + \sqrt{5}}{4} \times \frac{\sqrt{15} - \sqrt{3}}{3} a_{12} = \frac{\sqrt{15} + \sqrt{3}}{6} a_{12}$$

Desarrollo del cálculo anterior:  $\boxed{b_1} = \frac{3 + \sqrt{5}}{4} \times \frac{\sqrt{15} - \sqrt{3}}{3} a_{12} =$

$$= \frac{(3 + \sqrt{5})(\sqrt{15} - \sqrt{3})}{12} a_{12} = \frac{3\sqrt{15} + \sqrt{75} - 3\sqrt{3} - \sqrt{15}}{12} a_{12} = \frac{2\sqrt{15} + 5\sqrt{3} - 3\sqrt{3}}{12} a_{12} =$$

$$= \frac{2\sqrt{15} + 2\sqrt{3}}{12} a_{12} = \boxed{\frac{\sqrt{15} + \sqrt{3}}{6} a_{12}}$$

Radio "c<sub>12</sub>" de la esfera inscrita en el cubo.

Se deduce de la fórm. 32, lám. 4

$$c_{12} = \sqrt{\frac{11\sqrt{5} + 25}{40}} \cdot \frac{1}{2} a_{12} = \sqrt{\frac{11\sqrt{5} + 25}{40}} \times \frac{\sqrt{15} - \sqrt{3}}{3} a_{12} = \sqrt{\frac{5 + 2\sqrt{5}}{15}} a_{12}$$

Desarrollo del cálculo anterior:  $\boxed{c_{12}} = \sqrt{\frac{11\sqrt{5} + 25}{40}} \times \frac{\sqrt{15} - \sqrt{3}}{3} a_{12} =$

$$= \sqrt{\frac{(11\sqrt{5} + 25)(\sqrt{15} - \sqrt{3})^2}{40 \times 3^2}} a_{12} = \sqrt{\frac{(11\sqrt{5} + 25)(15 + 3 - 2\sqrt{45})}{40 \times 9}} a_{12} =$$

$$= \sqrt{\frac{(11\sqrt{5} + 25)(18 - 6\sqrt{5})}{40 \times 9}} a_{12} = \sqrt{\frac{(35 + 11\sqrt{5})(6 - 2\sqrt{5})}{40 \times 3}} a_{12} =$$

$$= \sqrt{\frac{150 + 66\sqrt{5} - 50\sqrt{5} - 22 \times 5}{40 \times 3}} a_{12} = \sqrt{\frac{40 + 16\sqrt{5}}{40 \times 3}} a_{12} = \boxed{\sqrt{\frac{5 + 2\sqrt{5}}{15}} a_{12}}$$



Radio " $d_{12}$ " de la circunferencia circunscrita al polígono regular de una cara del mismo.

Se deduce de la fórm. 33, lám. 4

$$d_{12} = \sqrt{\frac{5+\sqrt{5}}{10}} \quad l_{12} = \sqrt{\frac{5+\sqrt{5}}{10}} \times \frac{\sqrt{15}-\sqrt{3}}{3} a_{12} = \sqrt{\frac{10-2\sqrt{5}}{15}} a_{12}$$

Desarrollo del cálculo anterior:  $\boxed{d_{12}} = \sqrt{\frac{5+\sqrt{5}}{10}} \times \frac{\sqrt{15}-\sqrt{3}}{3} a_{12} =$

$$= \sqrt{\frac{(5+\sqrt{5})(\sqrt{15}-\sqrt{3})^2}{10 \times 3^2}} a_{12} = \sqrt{\frac{(5+\sqrt{5})(15+3-2\sqrt{45})}{10 \times 3^2}} a_{12} =$$

$$= \sqrt{\frac{(5+\sqrt{5})(18-6\sqrt{5})}{10 \times 9}} a_{12} = \sqrt{\frac{(5+\sqrt{5})(3-\sqrt{5})}{15}} a_{12} = \sqrt{\frac{15+3\sqrt{5}-5\sqrt{5}-5}{15}} a_{12} =$$

$$= \boxed{\sqrt{\frac{10-2\sqrt{5}}{15}} a_{12}}$$

Radio " $k_{12}$ " de la circunferencia inscrita al polígono regular de una cara del mismo (apotema).

Se deduce de la fórmula 39, lám. 4

$$k_{12} = \sqrt{\frac{5+2\sqrt{5}}{20}} \quad l_{12} = \sqrt{\frac{5+2\sqrt{5}}{20}} \times \frac{\sqrt{15}-\sqrt{3}}{3} a_{12} = \sqrt{\frac{5+\sqrt{5}}{30}} a_{12}$$

Desarrollo del cálculo anterior:  $\boxed{k_{12}} = \sqrt{\frac{5+2\sqrt{5}}{20}} \times \frac{\sqrt{15}-\sqrt{3}}{3} a_{12} =$

$$= \sqrt{\frac{(5+2\sqrt{5})(\sqrt{15}-\sqrt{3})^2}{20 \times 3^2}} a_{12} = \sqrt{\frac{(5+2\sqrt{5})(15+3-2\sqrt{45})}{20 \times 9}} a_{12} =$$

$$= \sqrt{\frac{(5+2\sqrt{5})(18-6\sqrt{5})}{20 \times 9}} a_{12} = \sqrt{\frac{(5+2\sqrt{5})(3-\sqrt{5})}{10 \times 3}} a_{12} =$$

The first part of the book discusses the history of the region and the role of the government in the development of the area. It also covers the economic and social conditions of the region at the time of the study. The second part of the book focuses on the political and administrative structure of the region, including the role of the local government and the central government. The third part of the book discusses the cultural and social aspects of the region, including the role of the community and the family. The fourth part of the book discusses the environmental and natural resources of the region, including the role of the government in the management of these resources. The fifth part of the book discusses the future of the region and the role of the government in the development of the area.

Chapter 2

The second part of the book discusses the political and administrative structure of the region, including the role of the local government and the central government. It also covers the economic and social conditions of the region at the time of the study. The third part of the book discusses the cultural and social aspects of the region, including the role of the community and the family. The fourth part of the book discusses the environmental and natural resources of the region, including the role of the government in the management of these resources. The fifth part of the book discusses the future of the region and the role of the government in the development of the area.



$$= \sqrt{\frac{15 + 6\sqrt{5} - 5\sqrt{5} - 10}{30}} a_{12} = \boxed{\sqrt{\frac{5 + \sqrt{5}}{30}} a_{12}}$$

Ángulo rectilíneo "2  $\varphi_{12}$ " del diedro del mismo

Se deduce de la fórmula 34, lám. 4

$$\text{sen } \varphi_{12} = \sqrt{\frac{5 + \sqrt{5}}{10}} \quad 2 \varphi_{12} = 116^\circ 33' 54,2''$$

Tomando como base los valores anteriores, deduciremos los siguientes del poliedro derivado.

Ángulo rectilíneo. "2  $\alpha_{12}$ " del diedro formado por dos caras contiguas del poliedro derivado, en una arista del dodecaedro dado.

Se deduce de la fórmula general [4] (ver lám. 25), sustituyendo en ella los valores particulares de este caso.

$$\frac{1}{2} \alpha_{12} = \frac{a_{12} k_{12}}{(k_{12})^2 - a_{12} c_{12} + (c_{12})^2} = \frac{a_{12} \times \sqrt{\frac{5 + \sqrt{5}}{30}} a_{12}}{\left(\sqrt{\frac{5 + \sqrt{5}}{30}} a_{12}\right)^2 - a_{12} \sqrt{\frac{5 + 2\sqrt{5}}{15}} a_{12} + \left(\sqrt{\frac{5 + 2\sqrt{5}}{15}} a_{12}\right)^2}$$

$$= 3 + \sqrt{\frac{15 + 3\sqrt{5}}{2}}$$

Desarrollo del cálculo anterior:  $\boxed{\frac{1}{2} \alpha_{12}} = \frac{\sqrt{\frac{5 + \sqrt{5}}{30}}}{\frac{5 + \sqrt{5}}{30} - \sqrt{\frac{5 + 2\sqrt{5}}{15}} + \frac{5 + 2\sqrt{5}}{15}}$

$$= \frac{\sqrt{\frac{5 + \sqrt{5}}{30}}}{\frac{5 + \sqrt{5} + 10 + 4\sqrt{5}}{30} - \sqrt{\frac{5 + 3\sqrt{5}}{15}}} = \frac{\sqrt{\frac{5 + \sqrt{5}}{30}}}{\frac{15 + 5\sqrt{5}}{30} - \sqrt{\frac{5 + 2\sqrt{5}}{15}}}$$

# Chapter 10

The first part of the chapter discusses the importance of maintaining accurate records of all transactions. This includes not only the amount of the transaction but also the date, the parties involved, and the purpose of the transaction. Proper record keeping is essential for the preparation of financial statements and for the detection of errors or fraud.

In the second part of the chapter, the author discusses the various methods used to collect and analyze data. These methods include surveys, interviews, and experiments. Each method has its own strengths and weaknesses, and the choice of method depends on the nature of the research question.

The third part of the chapter discusses the various methods used to analyze data. These methods include descriptive statistics, inferential statistics, and regression analysis. Each method is used to answer different types of research questions, and the choice of method depends on the nature of the data and the research question.

The final part of the chapter discusses the various methods used to present data. These methods include tables, graphs, and charts. Each method is used to present different types of data, and the choice of method depends on the nature of the data and the research question.

$$= \frac{\sqrt{30(5+\sqrt{5})}}{(\sqrt{15+5\sqrt{5}}) - \sqrt{60(5+2\sqrt{5})}} = \frac{\sqrt{30(5+\sqrt{5})}}{5(3+\sqrt{5}) - 2\sqrt{15(5+2\sqrt{5})}} =$$

$$= \frac{\sqrt{30(5+\sqrt{5})} \times [5(3+\sqrt{5}) + 2\sqrt{15(5+2\sqrt{5})}]}{25 \times (9+5+6\sqrt{5}) - 4 \times 15 \times (5+2\sqrt{5})} =$$

$$= \frac{\sqrt{30(5+\sqrt{5})} \times [5(3+\sqrt{5}) + 2\sqrt{15(5+2\sqrt{5})}]}{350 + 150\sqrt{5} - 300 - 120\sqrt{5}} =$$

$$= \frac{\sqrt{30(5+\sqrt{5})} \times [5(3+\sqrt{5}) + 2\sqrt{15(5+2\sqrt{5})}]}{50 + 30\sqrt{5}} =$$

$$= \frac{\sqrt{30(5+\sqrt{5})} \times (3\sqrt{5}-5) \times [5(3+\sqrt{5}) + 2\sqrt{15(5+2\sqrt{5})}]}{10(3\sqrt{5}+5)(3\sqrt{5}-5)} =$$

$$= \frac{\sqrt{30(5+\sqrt{5})}(3\sqrt{5}-5)^2 \times [5(3+\sqrt{5}) + 2\sqrt{15(5+2\sqrt{5})}]}{10(45-25)} =$$

$$= \frac{\sqrt{30(5+\sqrt{5})}(45+25-30\sqrt{5}) \times [5(3+\sqrt{5}) + 2\sqrt{15(5+2\sqrt{5})}]}{10 \times 20} =$$

$$= \frac{\sqrt{30(5+\sqrt{5})}(70-30\sqrt{5}) \times [5(3+\sqrt{5}) + 2\sqrt{15(5+2\sqrt{5})}]}{10 \times 20} =$$

$$= \frac{\sqrt{300(5+\sqrt{5})}(7-3\sqrt{5}) \times [5(3+\sqrt{5}) + 2\sqrt{15(5+2\sqrt{5})}]}{10 \times 20} =$$

$$= \frac{\sqrt{3(35+7\sqrt{5}-15\sqrt{5}-15)} \times [5(3+\sqrt{5}) + 2\sqrt{15(5+2\sqrt{5})}]}{20} =$$

1	1000	1000	1000
2	1000	1000	1000
3	1000	1000	1000
4	1000	1000	1000
5	1000	1000	1000
6	1000	1000	1000
7	1000	1000	1000
8	1000	1000	1000
9	1000	1000	1000
10	1000	1000	1000
11	1000	1000	1000
12	1000	1000	1000
13	1000	1000	1000
14	1000	1000	1000
15	1000	1000	1000
16	1000	1000	1000
17	1000	1000	1000
18	1000	1000	1000
19	1000	1000	1000
20	1000	1000	1000
21	1000	1000	1000
22	1000	1000	1000
23	1000	1000	1000
24	1000	1000	1000
25	1000	1000	1000
26	1000	1000	1000
27	1000	1000	1000
28	1000	1000	1000
29	1000	1000	1000
30	1000	1000	1000
31	1000	1000	1000
32	1000	1000	1000
33	1000	1000	1000
34	1000	1000	1000
35	1000	1000	1000
36	1000	1000	1000
37	1000	1000	1000
38	1000	1000	1000
39	1000	1000	1000
40	1000	1000	1000
41	1000	1000	1000
42	1000	1000	1000
43	1000	1000	1000
44	1000	1000	1000
45	1000	1000	1000
46	1000	1000	1000
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48	1000	1000	1000
49	1000	1000	1000
50	1000	1000	1000
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56	1000	1000	1000
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89	1000	1000	1000
90	1000	1000	1000
91	1000	1000	1000
92	1000	1000	1000
93	1000	1000	1000
94	1000	1000	1000
95	1000	1000	1000
96	1000	1000	1000
97	1000	1000	1000
98	1000	1000	1000
99	1000	1000	1000
100	1000	1000	1000

$$= \frac{\sqrt{3(20-2\sqrt{5})} \times [5(3+\sqrt{5}) + 2\sqrt{15(5+2\sqrt{5})}]}{20} =$$

$$= \frac{\sqrt{3(5-2\sqrt{5})} \times [5(3+\sqrt{5}) + 2\sqrt{15(5+2\sqrt{5})}]}{10} =$$

$$= \frac{5\sqrt{3(5-2\sqrt{5})} \times (3+\sqrt{5}) + 2\sqrt{3(5-2\sqrt{5})} \times \sqrt{15(5+2\sqrt{5})}}{10} =$$

$$= \frac{5\sqrt{3(5-2\sqrt{5})}(3+\sqrt{5})^2 + 2\sqrt{45(5-2\sqrt{5})}(5+2\sqrt{5})}{10} =$$

$$= \frac{5\sqrt{3(5-2\sqrt{5})}(9+5+6\sqrt{5}) + 2 \times 3 \times \sqrt{5(25-20)}}{10} =$$

$$= \frac{5\sqrt{3(5-2\sqrt{5})}(14+6\sqrt{5}) + 30}{10} = \frac{\sqrt{3 \times 2(5-2\sqrt{5})}(7+3\sqrt{5}) + 6}{2} =$$

$$= \frac{\sqrt{6(35-14\sqrt{5}+15\sqrt{5}-30)} + 6}{2} = 3 + \sqrt{\frac{6(5+\sqrt{5})}{4}} = 3 + \sqrt{\frac{3(5+\sqrt{5})}{2}} =$$

$$= 3 + \sqrt{\frac{15+3\sqrt{5}}{2}}$$

El valor numérico de  $\alpha_{12}$  en grados sexagesimales, será:

$$\frac{1}{2} \alpha_{12} = 3 + \sqrt{\frac{15+3\sqrt{5}}{2}} = 6,2945564\dots; \quad \lg \operatorname{tg} \alpha_{12} = 0,7989657$$

$$\alpha_{12} = 80^{\circ} 58' 22,8''.$$

$$2\alpha_{12} = 161^{\circ} 56' 45,6''$$

El valor  $\alpha_{12} < 90^{\circ}$  nos demuestra la convexidad



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del poliedro derivado (ver lám. 25, "Consideraciones previas").

Altura "p" de una cara lateral de la pirámide recta formada en cada cara del dodecaedro dado (cara del poliedro derivado).

Se deduce de la fórmula general [5] (ver lám. 25), sustituyendo en ella los valores particulares de este caso.

$$p = \sqrt{(a_{12} - c_{12})^2 + (k_{12})^2} = \sqrt{\left[a_{12} - \left(\sqrt{\frac{5+2\sqrt{5}}{15}} a_{12}\right)\right]^2 + \left(\sqrt{\frac{5+\sqrt{5}}{30}} a_{12}\right)^2} =$$

$$= \sqrt{\frac{9+\sqrt{5}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}} a_{12} \quad \left( = \sqrt{\left(1 - \sqrt{\frac{5+2\sqrt{5}}{15}}\right)^2 + \frac{5+\sqrt{5}}{30}} a_{12} \right)$$

Desarrollo del cálculo anterior:  $\boxed{p} = \sqrt{\left[a_{12} - \left(\sqrt{\frac{5+2\sqrt{5}}{15}} a_{12}\right)\right]^2 + \left(\sqrt{\frac{5+\sqrt{5}}{30}} a_{12}\right)^2}$

$$= \sqrt{\left(1 - \sqrt{\frac{5+2\sqrt{5}}{15}}\right)^2 a_{12}^2 + \frac{5+\sqrt{5}}{30} a_{12}^2} = \sqrt{1 + \frac{5+2\sqrt{5}}{15} - 2\sqrt{\frac{5+2\sqrt{5}}{15}} + \frac{5+\sqrt{5}}{30}} a_{12}$$

$$= \sqrt{\frac{30+10+4\sqrt{5}+5+\sqrt{5}}{30} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}} a_{12} = \sqrt{\frac{45+5\sqrt{5}}{30} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}} a_{12} =$$

$$= \sqrt{\frac{9+\sqrt{5}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}} a_{12}$$

Arista lateral "q" de la pirámide recta regular, o lado igual del triángulo isósceles de una cara del poliedro derivado.

Se deduce de la fórmula general [6] (ver lám. 25) susti-



Buscando en ella los valores particulares de este caso.

$$q = \sqrt{(a_{12} - c_{12})^2 + (d_{12})^2} = \sqrt{\left[a_{12} - \left(\sqrt{\frac{5+2\sqrt{5}}{15}} a_{12}\right)\right]^2 + \left(\sqrt{\frac{10-2\sqrt{5}}{15}} a_{12}\right)^2} =$$

$$= \sqrt{2 - 2\sqrt{\frac{5+2\sqrt{5}}{15}}} a_{12} \quad \left( = \sqrt{\left(1 - \sqrt{\frac{5+2\sqrt{5}}{15}}\right)^2 + \frac{10-2\sqrt{5}}{15}} a_{12} \right)$$

Desarrollo del cálculo anterior:  $\boxed{q} = \sqrt{\left[a_{12} - \left(\sqrt{\frac{5+2\sqrt{5}}{15}} a_{12}\right)\right]^2 + \left(\sqrt{\frac{10-2\sqrt{5}}{15}} a_{12}\right)^2} =$

$$= \sqrt{\left(1 - \sqrt{\frac{5+2\sqrt{5}}{15}}\right)^2 a_{12}^2 + \frac{10-2\sqrt{5}}{15} a_{12}^2} = \sqrt{1 + \frac{5+2\sqrt{5}}{15} - 2\sqrt{\frac{5+2\sqrt{5}}{15}} + \frac{10-2\sqrt{5}}{15}} a_{12} =$$

$$= \sqrt{\frac{15+5+2\sqrt{5}+10-2\sqrt{5}}{15}} - 2 \cdot \sqrt{\frac{5+2\sqrt{5}}{15}} a_{12} = \boxed{\sqrt{2 - 2\sqrt{\frac{5+2\sqrt{5}}{15}}} a_{12}}$$

Segmento "t" que se obtiene al unir los extremos de dos lados consecutivos del polígono de una cara del dodecaedro dado.

Es la diagonal del pentágono regular de lado  $l_{12}$ ; su valor s/ la geometría racional, se obtiene

$$t = \frac{d_{12}}{2} \sqrt{10+2\sqrt{5}} = \frac{\sqrt{\frac{10-2\sqrt{5}}{15}} a_{12}}{2} \times \sqrt{10+2\sqrt{5}} = \frac{2\sqrt{3}}{3} a_{12}$$

Desarrollo del cálculo anterior:  $\boxed{t} = \sqrt{\frac{10-2\sqrt{5}}{15}} \times \frac{\sqrt{10+2\sqrt{5}}}{2} a_{12} =$

$$= \frac{\sqrt{(10-2\sqrt{5})(10+2\sqrt{5})}}{2\sqrt{15}} a_{12} = \frac{\sqrt{80}}{2\sqrt{15}} a_{12} = \frac{4\sqrt{5}}{2\sqrt{15}} a_{12} = 2\sqrt{\frac{5}{15}} a_{12} = \boxed{\frac{2\sqrt{3}}{3} a_{12}}$$

The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is equivalent to finding a function  $f(x)$  which satisfies the conditions

$$f(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$$

and

$$f(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$$

The second part of the paper is devoted to a detailed study of the properties of the function  $f(x)$ . It is shown that the function  $f(x)$  is continuous and that it satisfies the conditions

$$f(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$$

and

$$f(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$$

The third part of the paper is devoted to a study of the properties of the function  $f(x)$ . It is shown that the function  $f(x)$  is continuous and that it satisfies the conditions

$$f(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$$

and

$$f(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$$

The fourth part of the paper is devoted to a study of the properties of the function  $f(x)$ . It is shown that the function  $f(x)$  is continuous and that it satisfies the conditions

$$f(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$$

and

$$f(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$$



Ángulo rectilíneo del diedro " $2\gamma_{12}$ " formado por dos caras laterales contiguas en las aristas de la pirámide recta.

Se deduce de la fórmula general [7] (ver lám. 25), sustituyendo en ella los valores particulares de este caso.

$$\begin{aligned} \operatorname{sen} \gamma_{12} &= \frac{tg}{2 l_{12} p} = \frac{\frac{2\sqrt{3}}{3} a_{12} \times \sqrt{2 - 2\sqrt{\frac{5+2\sqrt{5}}{15}}} a_{12}}{2 \times \frac{\sqrt{15}-\sqrt{3}}{3} a_{12} \times \sqrt{\frac{9+\sqrt{5}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}} a_{12}} = \\ &= \frac{\sqrt{3}}{\sqrt{15}-\sqrt{3}} \times \frac{9}{p} = \frac{1+\sqrt{5}}{4} \times \frac{9}{p} = \frac{0,8090170 \times 0,6408518}{0,5323242} = 0,9739554.. \end{aligned}$$

$$\operatorname{sen} \gamma_{12} = \frac{1+\sqrt{5}}{4} \times \sqrt{\left[2 - 2\sqrt{\frac{5+2\sqrt{5}}{15}}\right] : \left[\frac{9+\sqrt{5}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}\right]}$$

$$tg \operatorname{sen} \gamma_{12} = tg 0,9739554 = 7,9885391$$

$$\gamma_{12} = 76^{\circ} 53' 41,4''$$

$$2\gamma_{12} = 153^{\circ} 47' 22,8''$$

Ángulo diedro " $\beta_{12}$ " formado por una cara lateral de la pirámide y su base.

Se deduce de la fórmula general [8] (ver lám. 25), sustituyendo en ella los valores particulares de este caso.

$$\operatorname{sen} \beta_{12} = \frac{a_{12} - c_{12}}{p} = \frac{a_{12} - \sqrt{\frac{5+2\sqrt{5}}{15}} a_{12}}{\sqrt{\frac{9+\sqrt{5}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}} a_{12}} = \frac{1 - \sqrt{\frac{5+2\sqrt{5}}{15}}}{\sqrt{\frac{9+\sqrt{5}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}}}$$

Q.1. A particle is moving with a constant velocity of 10 m/s. Calculate the distance travelled by the particle in 5 seconds.

Sol. Given: Velocity = 10 m/s, Time = 5 s. To find: Distance.

$$\text{Distance} = \text{Velocity} \times \text{Time}$$

$$= 10 \times 5 = 50 \text{ m}$$

Q.2. A car starts from rest and accelerates uniformly to a speed of 20 m/s in 10 seconds. Calculate the acceleration.

$$a = \frac{v - u}{t}$$

$$= \frac{20 - 0}{10} = 2 \text{ m/s}^2$$

Q.3. A ball is thrown vertically upwards with an initial velocity of 15 m/s. Calculate the maximum height reached by the ball.

Sol. Given: Initial velocity = 15 m/s. To find: Maximum height.

$$v^2 = u^2 + 2as$$

$$0 = 15^2 + 2(-10)s$$

$$s = \frac{15^2}{20} = 11.25 \text{ m}$$

El valor numérico de  $\beta_{12}$ , se obtiene como sigue:

$$\operatorname{sen} \beta_{12} = \frac{1 - 0,7946545\dots}{0,5323242\dots} \approx 0,3857527\dots$$

$$\lg. \operatorname{sen} \beta_{12} = \lg. 0,3857527 = \bar{7},5863090$$

$$\beta_{12} = \underline{22^{\circ} 41' 25,7''}$$

debiendo verificarse como comprobación (ver fórm. [11], lám. 25)

$$\begin{aligned} \alpha_{12} &= \varphi_{12} + \beta_{12} = 58^{\circ} 16' 57,7'' \\ &\quad + \underline{22^{\circ} 41' 25,7''} \\ \alpha_{12} &= \underline{\underline{80^{\circ} 58' 22,8''}} \end{aligned}$$

valor coincidente con el ya obtenido en este estudio

Radio "b<sub>2</sub>" de la esfera tangente a las aristas laterales de las pirámides rectas cuyas bases son caras del dodecaedro regular dado.

Se deduce de la fórmula general [9] (ver lám. 25), sustituyendo en ella los valores particulares de este caso.

$$b_2 = \sqrt{(a_{12})^2 - \frac{9^2}{4}} = \sqrt{1 - \left(3 - 2\sqrt{\frac{5+2\sqrt{5}}{15}}\right) : 4} \quad a_{12} = \sqrt{\left(1 + \sqrt{\frac{5+2\sqrt{5}}{15}}\right) : 2} a_{12}$$

Received of \_\_\_\_\_ the sum of \_\_\_\_\_

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Radio " $C_1$ " de la esfera inscrita en el poliedro derivado

Se deduce de la fórmula general [10] (ver lám. 25), substituyendo en ella los valores particulares de este caso.

$$C_1 = b_1 \operatorname{sen} \alpha_{12} \quad \text{en la que} \quad b_1 = \frac{\sqrt{15} + \sqrt{3}}{6} a_{12}, \quad 7$$

$$\operatorname{tg} \alpha_{12} = 3 + \sqrt{\frac{15 + 3\sqrt{5}}{2}};$$

de esta última se obtiene:

$$\operatorname{sen} \alpha_{12} = \frac{\operatorname{tg} \alpha_{12}}{\sqrt{1 + \operatorname{tg}^2 \alpha_{12}}} = \frac{3 + \sqrt{\frac{15 + 3\sqrt{5}}{2}}}{\sqrt{1 + \left(3 + \sqrt{\frac{15 + 3\sqrt{5}}{2}}\right)^2}} \quad \text{por lo que}$$

$$C_1 = \frac{\sqrt{15} + \sqrt{3}}{6} \times \frac{3 + \sqrt{\frac{15 + 3\sqrt{5}}{2}}}{\sqrt{1 + \left(3 + \sqrt{\frac{15 + 3\sqrt{5}}{2}}\right)^2}} a_{12} = 0,93 \, 41 \, 72 \, 4 \operatorname{sen} \alpha_{12} \quad 7_{12}$$

$$\operatorname{tg} \frac{C_1}{a_{12}} = \operatorname{tg} 0,93 \, 41 \, 72 \, 4 + \operatorname{tg} \operatorname{sen} 30^\circ 58' 22,8'' = \frac{7,970 \, 42 \, 70}{7,974 \, 58 \, 75} = \frac{7,965 \, 01 \, 45}{7,965 \, 01 \, 45}$$

$$C_1 = 0,92 \, 26 \, 02 \, 1 \times a_{12}$$

Este mismo valor se puede deducir, como comprobación, de la fórmula equivalente [10'] (ver lám. 25), en la que

$$C_1 = b_2 \operatorname{sen} \gamma_{12} = \sqrt{\left(1 + \sqrt{\frac{5 + 2\sqrt{5}}{15}}\right)} : 2 \times \operatorname{sen} \gamma_{12} \times a_{12}$$





cuyos valores numéricos son:

$$C_1 = 0,94\ 72\ 73\ 6... \times \text{sen } 46^\circ\ 53'\ 41,4'' \times a_{12} =$$

$$= 0,94\ 72\ 73\ 6... \times 0,97\ 39\ 55\ 4 \times a_{12} = 0,92\ 26\ 02\ 2... a_{12}$$

$$C_1 = \sqrt{\left(1 + \sqrt{\frac{5+2\sqrt{5}}{15}}\right) : 2} \times \frac{1+\sqrt{5}}{4} \times \sqrt{\left[2 - 2\sqrt{\frac{5+2\sqrt{5}}{15}}\right] : \left[\frac{7+\sqrt{5}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}\right]} \times a_{12}$$

valor coincidente con el ya obtenido anteriormente.

Área lateral "S" del poliedro derivado

Se obtiene como suma de las áreas laterales de las doce pirámides rectas de base pentagonal regular de lado " $l_{12}$ ", cuyas caras laterales son triángulos isósceles de base " $l_{12}$ " y altura " $p$ " ya determinados.

$$S = 12 \times 5 \times \frac{l_{12} \times p}{2} = 30 \times \frac{\sqrt{15} \cdot \sqrt{3}}{3} \times \sqrt{\frac{7+\sqrt{5}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}} \times (a_{12})^2$$

$$= 30 \times 0,71\ 36\ 44\ 2... \times 0,53\ 23\ 24\ 2... (a_{12})^2 = 11,39\ 67\ 02\ 3 (a_{12})^2$$

Volumen "V" del poliedro derivado

Se obtiene como suma del volumen del dodecaedro dado y de las doce pirámides formadas en sus caras.

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$$V = V_{12} + 12 \times \frac{S_5 \times h}{3}$$

siendo " $S_5$ " el área de una cara del dodecaedro, y " $h$ " la altura de la pirámide.

Para obtener  $V_{12}$  en función de  $a_{12}$ , ver lám. 4, fórm. 41 y 30 que nos dan

$$V_{12} = \frac{7\sqrt{5} + 15}{4} (l_{12})^3 = \frac{7\sqrt{5} + 15}{4} \times \left( \frac{4}{\sqrt{5} + \sqrt{3}} a_{12} \right)^3 = \frac{2(5\sqrt{3} + \sqrt{15})}{9} (a_{12})^3$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } \boxed{V_{12}} &= \frac{7\sqrt{5} + 15}{4} \times \left( \frac{4}{\sqrt{15} + \sqrt{3}} a_{12} \right)^3 = \\ &= \frac{7\sqrt{5} + 15}{4} \times \left( \frac{4(\sqrt{15} - \sqrt{3})}{15 - 3} a_{12} \right)^3 = \frac{7\sqrt{5} + 15}{4} \times \left( \frac{\sqrt{15} - \sqrt{3}}{3} a_{12} \right)^3 = \\ &= \frac{(7\sqrt{5} + 15) \times (15\sqrt{15} - 3 \times 15 \times \sqrt{3} + 3\sqrt{15} \times 3 - 3\sqrt{3})}{4 \times 27} (a_{12})^3 = \\ &= \frac{(7\sqrt{5} + 15) \times (24\sqrt{15} - 48\sqrt{3})}{4 \times 27} (a_{12})^3 = \frac{(7\sqrt{5} + 15)(\sqrt{15} - 2\sqrt{3}) \times 6}{27} (a_{12})^3 = \\ &= \frac{6 \times (7\sqrt{75} + 15\sqrt{15} - 14\sqrt{15} - 30\sqrt{3})}{27} (a_{12})^3 = \frac{6 \times (35\sqrt{3} - 30\sqrt{3} + \sqrt{15})}{27} (a_{12})^3 = \\ &= \boxed{\frac{2(5\sqrt{3} + \sqrt{15})}{9} (a_{12})^3} \end{aligned}$$

Por otra parte, tenemos que (ver lám. 4, fórm. 40)

$$S_5 = \frac{\sqrt{25 + 10\sqrt{5}}}{4} (l_{12})^2 = \frac{\sqrt{25 + 10\sqrt{5}}}{4} \times \left( \frac{\sqrt{15} - \sqrt{3}}{3} a_{12} \right)^2 = \frac{\sqrt{10} \times (5 - \sqrt{5})}{6} (a_{12})^2$$

Chapter 1: Introduction

Mathematics is a branch of science that deals with the study of numbers, shapes, and patterns. It is a universal language that helps us understand the world around us. Mathematics is used in many fields, including science, engineering, and business.

The purpose of this chapter is to introduce the basic concepts of mathematics and to provide a foundation for the study of more advanced topics. We will explore the history of mathematics and its applications in various fields.

Mathematics is a vast and beautiful subject that has fascinated humans for centuries. It is a tool that allows us to solve problems and make discoveries. We will learn how to use mathematical reasoning and logic to solve problems.

The first part of the chapter will focus on the basic concepts of numbers and arithmetic. We will learn how to add, subtract, multiply, and divide numbers. We will also learn about fractions and decimals.

The second part of the chapter will focus on the basic concepts of geometry. We will learn about points, lines, and shapes. We will also learn about the properties of these shapes and how to calculate their areas and volumes.

The third part of the chapter will focus on the basic concepts of algebra. We will learn about variables and equations. We will also learn how to solve problems using algebraic methods.

Mathematics is a powerful tool that helps us understand the world. It is a subject that is constantly evolving and expanding. We will learn how to use mathematics to solve problems and make discoveries.



$$\begin{aligned}
 \text{Desarrollo del cálculo anterior: } \boxed{S_5} &= \frac{\sqrt{25+10\sqrt{5}}}{4} \times \left( \frac{\sqrt{15}-\sqrt{3}}{3} a_{12} \right)^2 = \\
 &= \frac{\sqrt{25+10\sqrt{5}}}{4} \times \frac{15+3-2\sqrt{45}}{9} (a_{12})^2 = \frac{\sqrt{25+10\sqrt{5}}}{4} \times \frac{6(3-\sqrt{5})}{9} (a_{12})^2 = \\
 &= \frac{\sqrt{25+10\sqrt{5}} \times (3-\sqrt{5})}{6} (a_{12})^2 = \frac{\sqrt{(25+10\sqrt{5})(3-\sqrt{5})^2}}{6} (a_{12})^2 = \\
 &= \frac{\sqrt{(25+10\sqrt{5})(14-6\sqrt{5})}}{6} (a_{12})^2 = \frac{\sqrt{(25+10\sqrt{5})(7-3\sqrt{5}) \times 2}}{6} (a_{12})^2 = \\
 &= \frac{\sqrt{2 \times (175 + 70\sqrt{5} - 75\sqrt{5} - 150)}}{6} (a_{12})^2 = \frac{\sqrt{2(25-5\sqrt{5})}}{6} (a_{12})^2 = \\
 &= \boxed{\frac{\sqrt{10(5-\sqrt{5})}}{6} (a_{12})^2}
 \end{aligned}$$

por otra parte  $h = a_{12} - c_{12} = \left( 1 - \sqrt{\frac{5+2\sqrt{5}}{15}} \right) a_{12}$

y finalmente

$$\begin{aligned}
 V &= V_{12} + 12 \times \frac{S_5 \times h}{3} = \frac{2(5\sqrt{3} + \sqrt{15})}{9} (a_{12})^3 + 4 \times \frac{\sqrt{10(5-\sqrt{5})}}{6} \times h \times (a_{12})^2 \\
 &= \left[ \frac{2(5\sqrt{3} + \sqrt{15})}{9} + \frac{2\sqrt{10(5-\sqrt{5})}}{3} \times \left( 1 - \sqrt{\frac{5+2\sqrt{5}}{15}} \right) \right] (a_{12})^3 = \\
 &= \frac{2\sqrt{10(5-\sqrt{5})}}{3} (a_{12})^2
 \end{aligned}$$

Desarrollo del cálculo anterior:

$$\boxed{V} = \left[ \frac{2(5\sqrt{3} + \sqrt{15})}{9} + \frac{2\sqrt{10(5-\sqrt{5})}}{3} \times \left( 1 - \sqrt{\frac{5+2\sqrt{5}}{15}} \right) \right] (a_{12})^3 =$$

Page No.

Date

1. The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation  $f(x) = \frac{1}{x}$ . It is shown that  $f(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$  and a vertical asymptote at  $x = 0$ .

2. In the second part, we consider the function  $g(x) = \frac{1}{x^2}$ . It is shown that  $g(x)$  is also a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$  and a vertical asymptote at  $x = 0$ .

3. The third part of the paper is devoted to the study of the function  $h(x) = \frac{1}{x^3}$ . It is shown that  $h(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$  and a vertical asymptote at  $x = 0$ .

4. In the fourth part, we consider the function  $k(x) = \frac{1}{x^4}$ . It is shown that  $k(x)$  is also a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$  and a vertical asymptote at  $x = 0$ .

5. The fifth part of the paper is devoted to the study of the function  $l(x) = \frac{1}{x^5}$ . It is shown that  $l(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$  and a vertical asymptote at  $x = 0$ .

6. In the sixth part, we consider the function  $m(x) = \frac{1}{x^6}$ . It is shown that  $m(x)$  is also a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$  and a vertical asymptote at  $x = 0$ .

7. The seventh part of the paper is devoted to the study of the function  $n(x) = \frac{1}{x^7}$ . It is shown that  $n(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$  and a vertical asymptote at  $x = 0$ .

8. In the eighth part, we consider the function  $o(x) = \frac{1}{x^8}$ . It is shown that  $o(x)$  is also a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$  and a vertical asymptote at  $x = 0$ .

9. The ninth part of the paper is devoted to the study of the function  $p(x) = \frac{1}{x^9}$ . It is shown that  $p(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$  and a vertical asymptote at  $x = 0$ .

10. In the tenth part, we consider the function  $q(x) = \frac{1}{x^{10}}$ . It is shown that  $q(x)$  is also a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$  and a vertical asymptote at  $x = 0$ .

$$= \left[ \frac{2}{9} (5\sqrt{3} + \sqrt{15}) + \frac{2}{3} \sqrt{10(5-\sqrt{5})} - \frac{2}{3} \sqrt{10(5-\sqrt{5})} \times \sqrt{\frac{5+2\sqrt{5}}{15}} \right] (a_{12})^3 =$$

$$= \left[ \frac{2}{9} (5\sqrt{3} + \sqrt{15}) + \frac{2}{3} \sqrt{10(5-\sqrt{5})} - \frac{2}{3} \sqrt{\frac{10(5-\sqrt{5})(5+2\sqrt{5})}{15}} \right] (a_{12})^3 =$$

$$= \left[ \frac{2}{9} (5\sqrt{3} + \sqrt{15}) + \frac{2}{3} \sqrt{10(5-\sqrt{5})} - \frac{2}{3} \sqrt{\frac{10(25-5\sqrt{5}+10\sqrt{5}-10)}{15}} \right] (a_{12})^3 =$$

$$= \left[ \frac{2}{9} (5\sqrt{3} + \sqrt{15}) + \frac{2}{3} \sqrt{10(5-\sqrt{5})} - \frac{2}{3} \sqrt{\frac{10(15-5\sqrt{5})}{15}} \right] (a_{12})^3 =$$

$$= \left[ \frac{2}{9} (5\sqrt{3} + \sqrt{15}) + \frac{2}{3} \sqrt{10(5-\sqrt{5})} - \frac{2}{3} \sqrt{\frac{10(3-\sqrt{5})}{3}} \right] (a_{12})^3 =$$

$$= \left[ \frac{2}{9} (5\sqrt{3} + \sqrt{15}) + \frac{2}{3} \sqrt{10(5-\sqrt{5})} - \frac{2}{3} \sqrt{\frac{10}{3} \times 3-\sqrt{5}} \right] (a_{12})^3 =$$

$$= \left[ \frac{2}{9} (5\sqrt{3} + \sqrt{15}) + \frac{2}{3} \sqrt{10(5-\sqrt{5})} - \frac{2}{3} \sqrt{\frac{10}{3}} \cdot \left( \sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}} \right) \right] (a_{12})^3 =$$

$$= \left[ \frac{2}{9} (5\sqrt{3} + \sqrt{15}) + \frac{2}{3} \sqrt{10(5-\sqrt{5})} - \frac{2}{3} \left( \sqrt{\frac{50}{6}} + \sqrt{\frac{10}{6}} \right) \right] (a_{12})^3 =$$

$$= \left[ \frac{2}{9} (5\sqrt{3} + \sqrt{15}) + \frac{2}{3} \sqrt{10(5-\sqrt{5})} - \frac{6}{9} \left( \sqrt{\frac{50}{6}} + \sqrt{\frac{10}{6}} \right) \right] (a_{12})^3 =$$

$$= \left[ \frac{2}{9} (5\sqrt{3} + \sqrt{15}) + \frac{2}{3} \sqrt{10(5-\sqrt{5})} - \frac{1}{9} \left( \sqrt{300} + \sqrt{60} \right) \right] (a_{12})^3 =$$

Handwritten text in a script, likely Tamil, arranged in approximately 12 horizontal lines within a rectangular frame.

$$= \left[ \frac{2}{9} (5\sqrt{3} + \sqrt{15}) + \frac{2}{3} \sqrt{10(5-\sqrt{5})} - \frac{1}{9} (10\sqrt{3} + 2\sqrt{15}) \right] (a_{12})^3 =$$

$$= \left[ \frac{2}{9} (5\sqrt{3} + \sqrt{15}) + \frac{2}{3} \sqrt{10(5-\sqrt{5})} - \frac{2}{9} (5\sqrt{3} + 2\sqrt{15}) \right] (a_{12})^3 =$$

$$= \frac{2\sqrt{10(5-\sqrt{5})}}{3} (a_{12})^3$$



1. *Phragmites australis* (Cav.) Rostk Schmidt

2. *Scirpus americanus* (L.) Link

3. *Eleocharis acicularis* (L.) Rostk Schmidt

4. *Sagittaria arifolia* (L.) Link

5. *Alisma plantago-aquatica* (L.) Rostk Schmidt

6. *Sparganium angustifolium* Michx

7. *Najas* (L.) Rostk Schmidt

8. *Chara* (L.) Rostk Schmidt

9. *Hydrocotyle* (L.) Rostk Schmidt

10. *Utricularia* (L.) Rostk Schmidt

En el cuadro sinóptico que damos a continuación, resumimos los resultados anteriores:

CUADRO SINÓPTICO

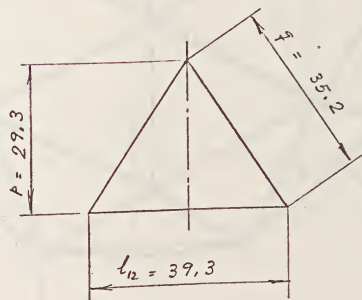
Magnitud	Valor exacto	Valor decimal aproximado
$f_{12}$	$\frac{\sqrt{15} - \sqrt{3}}{3} a_{12}$	0, 71 36 44... $a_{12}$
$b_1$	$\frac{\sqrt{15} + \sqrt{3}}{6} a_{12}$	0, 93 47 72... $a_{12}$
$b_2$	$\sqrt{\left(1 + \sqrt{\frac{5+2\sqrt{5}}{15}}\right)} : 2 \cdot a_{12}$	0, 94 72 74... $a_{12}$
$c_{12}$	$\frac{\sqrt{5+2\sqrt{5}}}{15} a_{12}$	0, 79 46 55... $a_{12}$
$C_1$	$\frac{\sqrt{15} + \sqrt{3}}{6} \times \left(3 + \sqrt{\frac{15+3\sqrt{5}}{2}}\right) : \sqrt{1 + \left(3 + \sqrt{\frac{15+3\sqrt{5}}{2}}\right)^2} = a_{12}$ $\sqrt{\left(1 + \sqrt{\frac{5+2\sqrt{5}}{15}}\right)} : 2 \times \left(\frac{1+\sqrt{5}}{4}\right) \times \sqrt{\left[2 - 2\sqrt{\frac{5+2\sqrt{5}}{15}}\right] : \left[\frac{9+\sqrt{5}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}\right]} a_{12}$	0, 92 26 02... $a_{12}$
$d_{12}$	$\frac{\sqrt{10 - 2\sqrt{5}}}{15} a_{12}$	0, 60 70 62... $a_{12}$
$k_{12}$	$\frac{\sqrt{5 + \sqrt{5}}}{30} a_{12}$	0, 49 11 24... $a_{12}$
$2\varphi_{12}$	$\text{sen } \varphi_{12} = \frac{\sqrt{5+\sqrt{5}}}{10} \quad \varphi_{12} = 58^\circ 16' 57,1''$	$\text{sen } \varphi_{12} = 0, 85 06 51$ $2\varphi_{12} = 116^\circ 33' 54,2''$
$2\alpha_{12}$	$\text{tg } \alpha_{12} = 3 + \sqrt{\frac{15+3\sqrt{5}}{2}} \quad \alpha_{12} = 80^\circ 58' 22,8''$	$\text{tg } \alpha_{12} = 6, 29 45 56$ $2\alpha_{12} = 161^\circ 56' 45,6''$
$2\gamma_{12}$	$\text{sen } \gamma_{12} = \frac{1+\sqrt{5}}{4} \times \sqrt{\left[2 - 2\sqrt{\frac{5+2\sqrt{5}}{15}}\right] : \left[\frac{9+\sqrt{5}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}\right]}$	$\text{sen } \gamma_{12} = 0, 97 39 55$ $2\gamma_{12} = 153^\circ 47' 22,8''$
$\beta_{12}$	$\text{sen } \beta_{12} = \left(1 - \sqrt{\frac{5+2\sqrt{5}}{15}}\right) : \sqrt{\frac{9+\sqrt{5}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}}$	$\text{sen } \beta_{12} = 0, 38 57 53$ $\beta_{12} = 22^\circ 41' 25,7''$
$p$	$\frac{\sqrt{9+\sqrt{5}}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}} a_{12}$	0, 53 23 24... $a_{12}$
$q$	$\sqrt{2 - 2\sqrt{\frac{5+2\sqrt{5}}{15}}} a_{12}$	0, 64 08 52... $a_{12}$
$t$	$\frac{2\sqrt{3}}{3} a_{12}$	1, 15 47 01... $a_{12}$
$S$	$30 \times \frac{\sqrt{15} - \sqrt{3}}{3} \times \frac{\sqrt{9+\sqrt{5}}}{6} - 2\sqrt{\frac{5+2\sqrt{5}}{15}} \times (a_{12})^2$	11, 39 67 02... $(a_{12})^2$
$V$	$\frac{2\sqrt{10(5-\sqrt{5})}}{3} (a_{12})^3$	3, 50 48 74... $(a_{12})^3$



FIGURA CORPÓREA

Se obtiene por acoplamiento de 60 caras iguales en forma de triángulos isósceles, de base  $l_{12} = 39,3$  mm y altura  $p = 29,3$  mm; en este triángulo el lado "q" tiene el valor  $q = 35,2$  mm (comprobación).

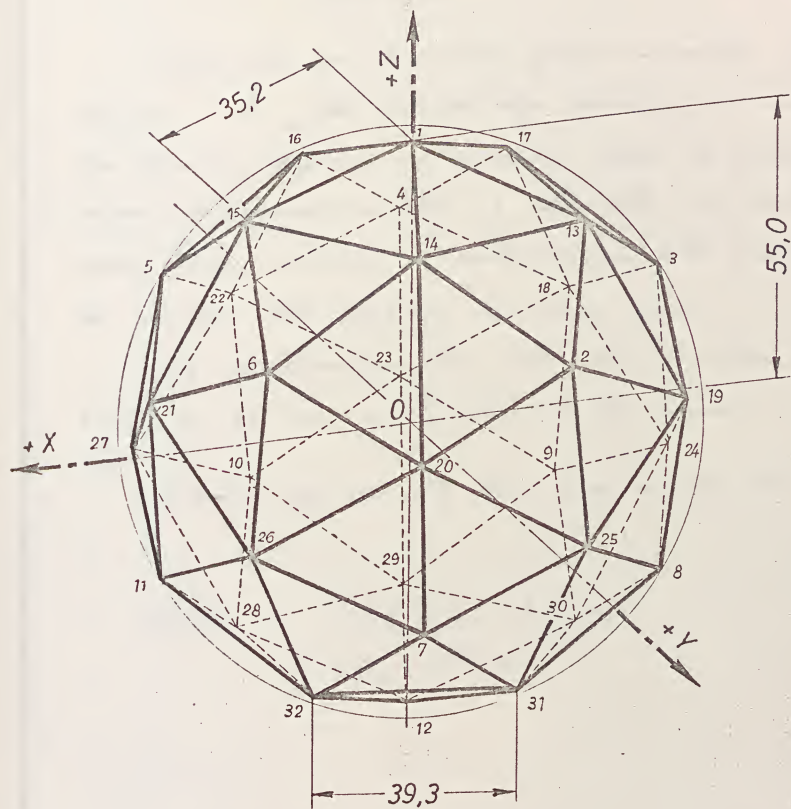
Para obtener este poliedro se formarán previamente 12 pirámides rectas de base pentagonal regular de lado " $l_{12}$ ", cuyas caras laterales son 5 triángulos (ver figura), acoplados por su lado "q".



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*Poliedro derivado del dodecaedro regular*



Figure 1. A diagram of a spherical structure with a network of lines.

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro derivado de un icosaedro regular, obtenido al proyectar desde el centro de la esfera circunscrita a éste, y sobre ella, los centros de cada cara, uniendo a continuación estos puntos con los vértices del polígono de dicha cara.

Las coordenadas del centro de la esfera, son:  
 $O(72, 72, 85)$  mm y el radio de la misma, de 55 mm.

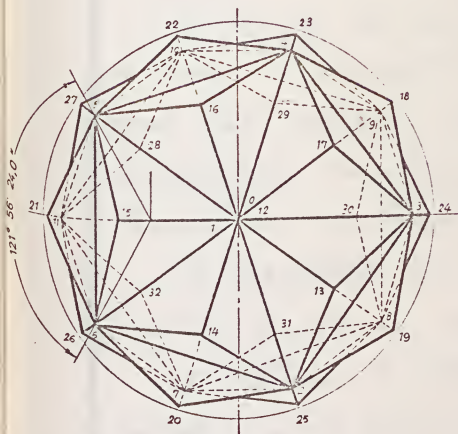
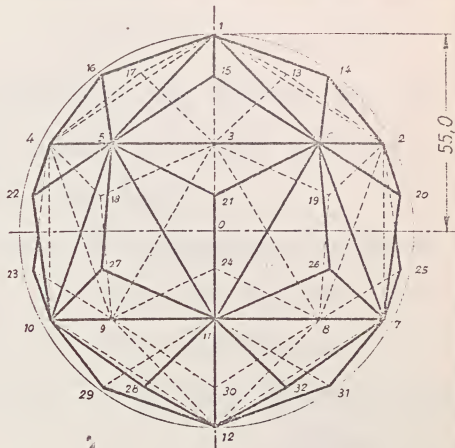
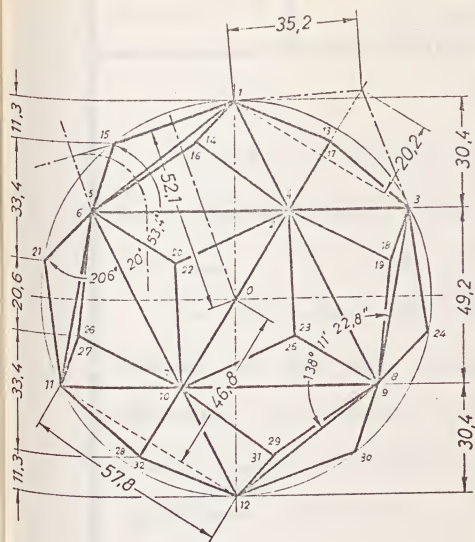
Dibujar en formato A3 y a escala 1:1.

DATOS:

$O(72, 72, 85)$  mm

$\rho_{20} = 55$  mm.





NUMERACIÓN DE VÉRTICES.

Icosaedro regular\_\_\_\_\_ 1 al 12

Proyecciones centros caras del mismo  
(vértices del dodecaedro conjugado).-- 13 al 32

### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro derivado de un icosaedro regular, obtenido al proyectar desde el centro de la esfera circunscrita a éste, y sobre ella, los centros de cada cara, uniendo a continuación estos puntos con los vértices del polígono de cada cara.

Las coordenadas del centro de la esfera, son: O (72, 72, 85) mm y el radio de la misma, de 55mm.

Dibujar en formato A3v y a escala 1:1.

	Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:						Curso
Alumno:						
Escala 1:1	Poliedro derivado del icosaedro regular					Lámina 29 Curso 19 -19





Al estudiar el ejercicio propuesto en la lámina 25, hemos obtenido unas deducciones previas de carácter general, comunes a los cinco poliedros regulares.

Las fórmulas allí deducidas las aplicaremos sucesivamente en este caso particular del icosaedro regular. El desarrollo del cálculo correspondiente a esta lámina, seguirá pues aquellas directrices, a las que haremos las oportunas referencias.

#### PROCESO GRÁFICO



En el caso del poliedro derivado del icosaedro regular, el proceso es inmediato, ya que sabemos que el conjugado del icosaedro es un dodecaedro regular, y esta representación ha sido ya realizada en el ejercicio de la lámina 24, cuyo proceso nos permite:

1º Representar el icosaedro regular dado, de vértices 1 al 12, inscrito en una esfera de 55 mm de radio.

2º Obtener los vértices 13 al 32 del dodecaedro conjugado inscrito en la misma esfera (estos vértices se han de corresponder con los 21 al 40 de la lámina 24).

3º Unir los vértices 21 al 40 con los correspondientes de cada cara del icosaedro dado.

Al terminar la representación del poliedro derivado,

The first part of the book is devoted to a general  
survey of the subject. It is divided into three  
main parts. The first part is devoted to the  
history of the subject. The second part is devoted  
to the theory of the subject. The third part is  
devoted to the practice of the subject.

The second part of the book is devoted to the  
theory of the subject. It is divided into two  
main parts. The first part is devoted to the  
theory of the subject. The second part is devoted  
to the practice of the subject.

The third part of the book is devoted to the  
practice of the subject. It is divided into two  
main parts. The first part is devoted to the  
theory of the subject. The second part is devoted  
to the practice of the subject.

podemos comprobar que éste es un poliedro cóncavo, de

$$C = 3 \times 20 = 60 \text{ caras (ver lám. 25, fórm. [1])}; \text{ de}$$

$$V = 20 + 12 = 32 \text{ vértices (ver lám. 25, fórm. [2])}; \text{ y de}$$

$$A = 30 + 3 \times 20 = 90 \text{ aristas (ver lám. 25, fórm. [3])}.$$

La demostración de la concavidad de este poliedro la haremos analíticamente.

### PROCESO GRÁFICO-ANALÍTICO

Calculemos previamente los siguientes valores deducidos de ejercicios anteriores, en función del radio  $a_{20}$  (dato) de la esfera circunscrita.

Número de caras "n" del icosaedro dado

$$n = 20$$

Radio " $a_{20}$ " de la esfera circunscrita al mismo (dato del ejercicio).

Lado " $l_{20}$ " del icosaedro dado

Se deduce de la fórm. 43, lám. 5

$$l_{20} = \frac{4}{\sqrt{10+2\sqrt{5}}} a_{20} = \sqrt{\frac{10-2\sqrt{5}}{5}} a_{20}$$

Desarrollo del cálculo anterior:  $\boxed{l_{20}} = \frac{4}{\sqrt{10+2\sqrt{5}}} a_{20} =$

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TEL. 773-936-5000



$$= \sqrt{\frac{16}{10+2\sqrt{5}}} a_{20} = \sqrt{\frac{8}{5+\sqrt{5}}} a_{20} = \sqrt{\frac{8(5-\sqrt{5})}{20}} a_{20} = \sqrt{\frac{2(5-\sqrt{5})}{5}} a_{20}$$

$$= \boxed{\sqrt{\frac{10-2\sqrt{5}}{5}} a_{20}}$$

Radio "b<sub>1</sub>" de la esfera tangente a las aristas del poliedro regular dado

Se deduce de la fórm. 44, lám. 5

$$b_1 = b_{20} = \frac{1+\sqrt{5}}{4} l_{20} = \frac{1+\sqrt{5}}{4} \times \sqrt{\frac{2(5-\sqrt{5})}{5}} a_{20} = \sqrt{\frac{5+\sqrt{5}}{10}} a_{20}$$

Desarrollo del cálculo anterior:  $\boxed{b_1} = \frac{1+\sqrt{5}}{4} \times \sqrt{\frac{2(5-\sqrt{5})}{5}} a_{20} =$

$$= \sqrt{\frac{2(5-\sqrt{5})(1+\sqrt{5})^2}{16 \times 5}} a_{20} = \sqrt{\frac{(5-\sqrt{5})(6+2\sqrt{5})}{8 \times 5}} a_{20} = \sqrt{\frac{(5-\sqrt{5})(3+\sqrt{5})}{20}} a_{20} =$$

$$= \sqrt{\frac{15-3\sqrt{5}+5\sqrt{5}-5}{20}} a_{20} = \sqrt{\frac{10+2\sqrt{5}}{20}} a_{20} = \boxed{\sqrt{\frac{5+\sqrt{5}}{10}} a_{20}}$$

Radio "c<sub>20</sub>" de la esfera inscrita en el mismo

Se deduce de la fórm. 45, lám. 5.

$$c_{20} = \frac{3\sqrt{3}+\sqrt{15}}{12} l_{20} = \frac{3\sqrt{3}+\sqrt{15}}{12} \times \sqrt{\frac{2(5-\sqrt{5})}{5}} a_{20} = \sqrt{\frac{5+2\sqrt{5}}{15}} a_{20}$$

Desarrollo del cálculo anterior:  $\boxed{c_{20}} = \frac{3\sqrt{3}+\sqrt{15}}{12} \times \sqrt{\frac{2(5-\sqrt{5})}{5}} a_{20} =$

$$= \sqrt{\frac{2(5-\sqrt{5})(3\sqrt{3}+\sqrt{15})^2}{12 \times 12 \times 5}} a_{20} = \sqrt{\frac{(5-\sqrt{5})(27+15+6\sqrt{45})}{6 \times 12 \times 5}} a_{20} =$$

Introduction

The purpose of this study is to investigate the effects of the proposed system on the performance of the system.

The results of the study show that the proposed system has a significant positive effect on the performance of the system.

The study also shows that the proposed system has a significant positive effect on the performance of the system.

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$$= \sqrt{\frac{(5-\sqrt{5})(42+18\sqrt{5})}{6 \times 12 \times 5}} a_{20} = \sqrt{\frac{(5-\sqrt{5})(7+3\sqrt{5})}{12 \times 5}} a_{20} = \sqrt{\frac{35-7\sqrt{5}+15\sqrt{5}-15}{12 \times 5}} a_{20}$$

$$= \sqrt{\frac{20+8\sqrt{5}}{12 \times 5}} a_{20} = \boxed{\sqrt{\frac{5+2\sqrt{5}}{15}} a_{20}}$$

Radio "d<sub>20</sub>" de la circunferencia circunscrita al polígono regular de una cara del mismo.

Se deduce de la fórm. 46, lám. 5

$$d_{20} = \frac{\sqrt{3}}{3} l_{20} = \frac{\sqrt{3}}{3} \times \sqrt{\frac{2(5-\sqrt{5})}{5}} a_{20} = \sqrt{\frac{2(5-\sqrt{5})}{15}} a_{20} = \sqrt{\frac{10-2\sqrt{5}}{15}} a_{20}$$

Radio "k<sub>20</sub>" de la circunferencia inscrita al polígono regular de una cara del mismo (apotema).

Se deduce de la fórm. 53, lám. 5.

$$k_{20} = \frac{\sqrt{3}}{6} l_{20} = \frac{\sqrt{3}}{6} \times \sqrt{\frac{2(5-\sqrt{5})}{5}} a_{20} = \sqrt{\frac{2(5-\sqrt{5})}{12 \times 5}} a_{20} = \sqrt{\frac{5-\sqrt{5}}{30}} a_{20}$$

Ángulo rectilíneo "2 φ<sub>20</sub>" del diedro del mismo.

Se deduce de la fórm. 47, lám. 5

$$\text{sen } \varphi_{20} = \frac{\sqrt{15} + \sqrt{3}}{6} \quad 2 \varphi_{20} = 138^{\circ} 11' 22,8''$$

Tomando como base los valores anteriores, deduciremos los siguientes del poliedro derivado

Main body of handwritten text, consisting of several lines of script, likely in Urdu or Persian, covering the majority of the page.

Ángulo rectilíneo "I  $\alpha_{20}$ " del diedro formado por dos caras contiguas del poliedro derivado, en una arista del icosaedro. dado.

Se deduce de la fórmula general [4] (ver lám. 25), substituyendo en ella los valores particulares de este caso.

$$\begin{aligned} \frac{1}{2} \alpha_{20} &= \frac{a_{20} k_{20}}{(k_{20})^2 - a_{20} c_{20} + (c_{20})^2} = \frac{a_{20} \cdot \sqrt{\frac{5-\sqrt{5}}{30}} a_{20}}{\left(\sqrt{\frac{5-\sqrt{5}}{30}} a_{20}\right)^2 - a_{20} \sqrt{\frac{5+2\sqrt{5}}{15}} a_{20} + \left(\sqrt{\frac{5+2\sqrt{5}}{15}} a_{20}\right)^2} \\ &= - \frac{\sqrt{6(5-\sqrt{5})} + 2\sqrt{5}}{2} \end{aligned}$$

Desarrollo del cálculo anterior:

$$\begin{aligned} \frac{1}{2} \alpha_{20} &= \frac{a_{20} \sqrt{\frac{5-\sqrt{5}}{30}} \cdot a_{20}}{\left(\sqrt{\frac{5-\sqrt{5}}{30}} a_{20}\right)^2 - a_{20} \sqrt{\frac{5+2\sqrt{5}}{15}} a_{20} + \left(\sqrt{\frac{5+2\sqrt{5}}{15}} a_{20}\right)^2} \\ &= \frac{\sqrt{\frac{5-\sqrt{5}}{30}}}{\frac{5-\sqrt{5}}{30} - \sqrt{\frac{5+2\sqrt{5}}{15}} + \frac{5+2\sqrt{5}}{15}} = \frac{\sqrt{\frac{5-\sqrt{5}}{30}}}{\frac{5-\sqrt{5} + 10 + 4\sqrt{5}}{30} - \sqrt{\frac{5+2\sqrt{5}}{15}}} \\ &= \frac{\sqrt{30(5-\sqrt{5})}}{15 + 3\sqrt{5} - \sqrt{\frac{30^2(5+2\sqrt{5})}{15}}} = \frac{\sqrt{30(5-\sqrt{5})}}{3(5+\sqrt{5}) - \sqrt{60(5+2\sqrt{5})}} \\ &= \frac{\sqrt{30(5-\sqrt{5})}}{3(5+\sqrt{5}) - 2\sqrt{15(5+2\sqrt{5})}} = \frac{\sqrt{30(5-\sqrt{5})} \times [3(5+\sqrt{5}) + 2\sqrt{15(5+2\sqrt{5})}]}{9 \times (25 + 5 + 10\sqrt{5}) - 4 \times 15 \times (5 + 2\sqrt{5})} \end{aligned}$$





$$\begin{aligned}
 & \frac{3\sqrt{30(5-\sqrt{5})(5+\sqrt{5})^2} + 2\sqrt{30 \times 15 \cdot (5-\sqrt{5})(5+2\sqrt{5})}}{270 + 90\sqrt{5} - 300 - 120\sqrt{5}} \\
 &= \frac{3\sqrt{30(25-5)(5+\sqrt{5})} + 2 \times 15\sqrt{2(25-5\sqrt{5}+10\sqrt{5}-10)}}{-30\sqrt{5} - 30} \\
 &= \frac{3\sqrt{30 \times 20 \times (5+\sqrt{5})} + 30\sqrt{2(15+5\sqrt{5})}}{-(30\sqrt{5} + 30)} = -\frac{3 \times 10\sqrt{6(5+\sqrt{5})} + 30\sqrt{10(3+\sqrt{5})}}{30(\sqrt{5} + 1)} \\
 &= -\frac{\sqrt{6(5+\sqrt{5})} + \sqrt{10(3+\sqrt{5})}}{\sqrt{5} + 1} = -\frac{\sqrt{6(5+\sqrt{5})(\sqrt{5}-1)^2} + \sqrt{10(3+\sqrt{5})(\sqrt{5}-1)^2}}{4} \\
 &= -\frac{\sqrt{6(5+\sqrt{5})(6-2\sqrt{5})} + \sqrt{10(3+\sqrt{5})(6-2\sqrt{5})}}{4} \\
 &= -\frac{2\sqrt{3(5+\sqrt{5})(3-\sqrt{5})} + 2\sqrt{5(3+\sqrt{5})(3-\sqrt{5})}}{4} \\
 &= -\frac{\sqrt{3(15+3\sqrt{5}-5\sqrt{5}-5)} + \sqrt{5(9-5)}}{2} = -\frac{\sqrt{3(10-2\sqrt{5})} + 2\sqrt{5}}{2} \\
 &= -\frac{\sqrt{6(5-\sqrt{5})} + 2\sqrt{5}}{2}
 \end{aligned}$$

El valor numérico de  $\alpha_{20}$  en grados sexagesimales, será:

$$\begin{aligned}
 \frac{1}{2} \alpha_{20} &= -\frac{\sqrt{6(5-\sqrt{5})} + 2\sqrt{5}}{2} = -4,2722158 = \frac{1}{2}(\pi - \delta) = -\frac{1}{2}\delta \\
 \frac{1}{2}\delta &= 4,2722158
 \end{aligned}$$

I hereby certify that the above is a true and correct copy of the original as shown to me by the person or institution named above.

In witness whereof, I have hereunto set my hand and seal at the place and date above written.

Signed and sealed in presence of the undersigned witnesses.

The undersigned witnesses have hereunto set their hands and seals at the place and date above written.

In presence of the undersigned witnesses.

The undersigned witnesses have hereunto set their hands and seals at the place and date above written.

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$$t_9 \quad t_7 \quad \delta = t_9. 4, 27 \quad 22 \quad 15' 8'' = 0,6306 \quad 532$$

$$\delta = 76^\circ 49' 33,3''$$

$$\alpha_{20} = 180^\circ - 76^\circ 49' 33,3'' = 103^\circ 10' 26,7''$$

$$2 \quad \alpha_{20} = 206^\circ 20' 53,4''$$

El valor de  $\alpha_{20} > 90^\circ$  nos demuestra la concavidad del poliedro derivado (ver lám. 25 "Consideraciones previas").

Altura "p" de una cara lateral de la pirámide recta formada en cada cara del icosaedro dado (cara del poliedro derivado).

Se deduce de la fórmula general [5] (ver lám. 25), sustituyendo en ella los valores particulares de este caso.

$$p = \sqrt{(a_{20} - c_{20})^2 + (k_{20})^2} = \sqrt{\left[a_{20} - \left(\frac{\sqrt{5+2\sqrt{5}}}{15} a_{20}\right)\right]^2 + \left(\frac{\sqrt{5-\sqrt{5}}}{30} a_{20}\right)^2} =$$

$$= \sqrt{\left(1 - \frac{\sqrt{5+2\sqrt{5}}}{15}\right)^2 + \frac{5-\sqrt{5}}{30}} \times a_{20} = \sqrt{\frac{15+\sqrt{5}}{10} - 2 \frac{\sqrt{5+2\sqrt{5}}}{15}} a_{20}$$

Desarrollo del cálculo anterior:  $\boxed{p} = \sqrt{\left(1 - \frac{\sqrt{5+2\sqrt{5}}}{15}\right)^2 + \frac{5-\sqrt{5}}{30}} a_{20} =$

$$= \sqrt{1 + \frac{5+2\sqrt{5}}{15} - 2 \frac{\sqrt{5+2\sqrt{5}}}{15} + \frac{5-\sqrt{5}}{30}} a_{20} = \sqrt{\frac{30+10+4\sqrt{5}+5-\sqrt{5}}{30} - 2 \frac{\sqrt{5+2\sqrt{5}}}{15}} a_{20} =$$

$$= \sqrt{\frac{45+3\sqrt{5}}{30} - 2 \frac{\sqrt{5+2\sqrt{5}}}{15}} a_{20} = \boxed{\sqrt{\frac{15+\sqrt{5}}{10} - 2 \frac{\sqrt{5+2\sqrt{5}}}{15}}} a_{20}$$





Arista lateral "q" de la pirámide recta regular, o lado igual del triángulo isósceles de una cara del poliedro derivado.

Se deduce de la fórmula general [6] (ver lám. 25), sustituyendo en ella los valores particulares de este caso.

$$q = \sqrt{(a_{20} - c_{20})^2 + (d_{20})^2} = \sqrt{\left[a_{20} - \left(\frac{5+2\sqrt{5}}{15} a_{20}\right)\right]^2 + \left(\frac{10-2\sqrt{5}}{15} a_{20}\right)^2} =$$

$$= \sqrt{\left(1 - \frac{5+2\sqrt{5}}{15}\right)^2 + \frac{10-2\sqrt{5}}{15}} a_{20} = \sqrt{2 - 2\frac{5+2\sqrt{5}}{15}} a_{20}$$

Desarrollo del cálculo anterior:  $q = \sqrt{\left[a_{20} - \left(\frac{5+2\sqrt{5}}{15} a_{20}\right)\right]^2 + \left(\frac{10-2\sqrt{5}}{15} a_{20}\right)^2} =$

$$= \sqrt{\left(1 - \frac{5+2\sqrt{5}}{15}\right)^2 (a_{20})^2 + \frac{10-2\sqrt{5}}{15} (a_{20})^2} = \sqrt{1 + \frac{5+2\sqrt{5}}{15} - 2\frac{5+2\sqrt{5}}{15} + \frac{10-2\sqrt{5}}{15}} a_{20}$$

$$= \sqrt{\frac{15+5+2\sqrt{5}+10-2\sqrt{5}}{15} - 2\frac{5+2\sqrt{5}}{15}} a_{20} = \sqrt{2 - 2\frac{5+2\sqrt{5}}{15}} a_{20}$$

Segmento "t" que se obtiene al unir los extremos de dos lados consecutivos del polígono de una cara del icosaedro dado.

Es el tercer lado del triángulo equilátero de una cara, o sea

$$t = l_{20} = \frac{\sqrt{10-2\sqrt{5}}}{5} a_{20}$$



The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined on the interval  $[0, 1]$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula:

$$f'(x) = \frac{1}{1+x^2}$$

which is valid for all  $x \in [0, 1]$ . The function  $f(x)$  is also shown to be concave down on this interval.

The second part of the paper is devoted to the study of the function  $g(x)$  defined on the interval  $[0, 1]$ . It is shown that  $g(x)$  is continuous and differentiable on this interval. The derivative of  $g(x)$  is given by the formula:

$$g'(x) = \frac{1}{1+x^2}$$

which is valid for all  $x \in [0, 1]$ . The function  $g(x)$  is also shown to be concave down on this interval.

The third part of the paper is devoted to the study of the function  $h(x)$  defined on the interval  $[0, 1]$ . It is shown that  $h(x)$  is continuous and differentiable on this interval. The derivative of  $h(x)$  is given by the formula:

$$h'(x) = \frac{1}{1+x^2}$$

which is valid for all  $x \in [0, 1]$ . The function  $h(x)$  is also shown to be concave down on this interval.

Ángulo rectilíneo del diedro "2  $\gamma_{20}$ " formado por dos caras laterales contiguas en las aristas de la pirámide recta.

Se deduce de la fórmula general [7] (ver lám. 25), sustituyendo en ella los valores particulares de este caso

$$\begin{aligned} \operatorname{sen} \gamma_{20} &= \frac{t q}{2 h_{20} p} = \frac{h_{20} q}{2 h_{20} p} = \frac{1}{2} \times \frac{q}{p} = \frac{1}{2} \times \frac{\sqrt{2 - 2 \sqrt{\frac{5+2\sqrt{5}}{15}}} a_{20}}{\sqrt{\frac{15+\sqrt{5}}{10}} - 2 \sqrt{\frac{5+2\sqrt{5}}{15}}} a_{20}} \\ &= \frac{0,64 \ 08 \ 51 \ 8...}{2 \times 0,36 \ 64 \ 66 \ 7...} = \frac{0,64 \ 08 \ 51 \ 8...}{0,73 \ 29 \ 33 \ 4...} = 0,87 \ 43 \ 65 \ 7... \end{aligned}$$

$$\operatorname{sen} \gamma_{20} = \sqrt{\left[ 2 - 2 \sqrt{\frac{5+2\sqrt{5}}{15}} \right] : \left[ 4 \cdot \left( \frac{15+\sqrt{5}}{10} - 2 \sqrt{\frac{5+2\sqrt{5}}{15}} \right) \right]}$$

$$\lg \operatorname{sen} \gamma_{20} = \lg 0,87 \ 43 \ 65 \ 7 = \bar{1},9416 \ 931$$

$$\gamma_{20} = 60^{\circ} \ 58' \ 12,0''$$

$$2 \gamma_{20} = 121^{\circ} \ 56' \ 24,0''$$

Ángulo diedro " $\beta_{20}$ " formado por una cara lateral de la pirámide y su base.

Se deduce de la fórmula general [8] (ver lám. 25), sustituyendo en ella los valores particulares de este caso

$$\operatorname{sen} \beta_{20} = \frac{a_{20} - c_{20}}{p} = \frac{a_{20} - \sqrt{\frac{5+2\sqrt{5}}{15}} a_{20}}{\sqrt{\frac{15+\sqrt{5}}{10}} - 2 \sqrt{\frac{5+2\sqrt{5}}{15}}} a_{20}} = \frac{1 - \sqrt{\frac{5+2\sqrt{5}}{15}}}{\sqrt{\frac{15+\sqrt{5}}{10}} - 2 \sqrt{\frac{5+2\sqrt{5}}{15}}}$$

The following is a list of the names of the persons who have been  
 named in the report of the committee on the subject of the  
 proposed amendment to the constitution of the State of New York.

The names of the persons who have been named in the report of the committee on the subject of the proposed amendment to the constitution of the State of New York are as follows:

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El valor numérico de  $\beta_{20}$ , se obtiene a continuación:

$$\operatorname{sen} \beta_{20} = \frac{1 - 0,79\ 46\ 54\ 5\dots}{0,36\ 64\ 66\ 7\dots} = \frac{0,20\ 53\ 45\ 5\dots}{0,36\ 64\ 66\ 7\dots} = 0,56\ 03\ 38\ 8\dots$$

$$\lg \operatorname{sen} \beta_{20} = \lg 0,56\ 03\ 38\ 8 \quad \beta_{20} = 34^{\circ}\ 4'\ 45,3''$$

debiendo verificarse como comprobación (ver fórm. [11], lám. 25)

$$\begin{aligned} \alpha_{20} &= \psi_{20} + \beta_{20} = 69^{\circ}\ 5'\ 41,4'' \\ &\quad + 34^{\circ}\ 4'\ 45,3'' \\ \hline \alpha_{20} &= 103^{\circ}\ 10'\ 26,7'' \end{aligned}$$

valor coincidente con el ya obtenido en este estudio.

Radio "b<sub>2</sub>" de la esfera tangente a las aristas laterales de las pirámides rectas cuyas bases son caras del icosaedro regular dado.

Se deduce de la fórmula general [9] (ver lám. 25), substituyendo en ella los valores particulares de este caso.

$$b_2 = \sqrt{(a_{20})^2 - \frac{9^2}{4}} = \sqrt{1 - (2 - 2\sqrt{\frac{5+2\sqrt{5}}{15}}):4} \quad a_{20} = \sqrt{4 + \sqrt{\frac{5+2\sqrt{5}}{15}}}:2 \cdot a_{20}$$

Radio "c<sub>1</sub>" de la esfera inscrita en el poliedro derivado

Se deduce de la fórmula general [10] (ver lám. 25)



The following is a list of the names of the persons who have been elected to the office of the President of the Association for the year 1911-12.

The names of the persons who have been elected to the office of the President of the Association for the year 1911-12 are as follows:

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The names of the persons who have been elected to the office of the President of the Association for the year 1911-12 are as follows:

sustituyendo en ella los valores particulares de este caso.

$$C_1 = b_1 \operatorname{sen} \alpha_{20}; \text{ en la que } b_1 = \sqrt{\frac{5+\sqrt{5}}{10}} a_{20} \quad ?$$

$$\operatorname{tg} \alpha_{20} = - \frac{\sqrt{6(5-\sqrt{5})} + 2\sqrt{5}}{2}$$

de esta última se obtiene

$$\operatorname{sen} \alpha_{20} = \frac{\operatorname{tg} \alpha_{20}}{\sqrt{1 + \operatorname{tg}^2 \alpha_{20}}} = \frac{- \frac{\sqrt{6(5-\sqrt{5})} + 2\sqrt{5}}{2}}{\sqrt{1 + \left( - \frac{\sqrt{6(5-\sqrt{5})} + 2\sqrt{5}}{2} \right)^2}} = -m$$

Pero estando  $\alpha_{20}$  comprendido entre  $\pi$  y  $\frac{\pi}{2}$ , el  $\operatorname{sen} \alpha_{20}$  será positivo, por lo que  $\operatorname{sen} \alpha_{20} = m$ , y finalmente

$$C_1 = \sqrt{\frac{5+\sqrt{5}}{10}} \times \frac{\sqrt{6(5-\sqrt{5})} + 2\sqrt{5}}{2\sqrt{1 + \left( \frac{\sqrt{6(5-\sqrt{5})} + 2\sqrt{5}}{2} \right)^2}} \times a_{20}$$

su valor numérico será:

$$\operatorname{tg} \frac{C_1}{a_{20}} = \operatorname{tg} \sqrt{\frac{5+\sqrt{5}}{10}} + \operatorname{tg} \operatorname{sen} 103^\circ 10' 26,7'' = \operatorname{tg} 0,8506508 +$$

$$+ \operatorname{tg} \operatorname{sen} (180^\circ - 103^\circ 10' 26,7'') = \begin{array}{r} 7,9297513 \\ 7,9884172 \\ \hline 7,9181685 \end{array}$$

$$C_1 = 0,8282635 \dots a_{20}$$

1. The first part of the document is a letter from the President of the United States to the Congress, dated September 17, 1787. The letter is signed by George Washington and is addressed to the members of the Congress. The letter is a copy of the original letter and is signed by George Washington.

2. The second part of the document is a letter from the President of the United States to the Congress, dated September 17, 1787. The letter is signed by George Washington and is addressed to the members of the Congress. The letter is a copy of the original letter and is signed by George Washington.

3. The third part of the document is a letter from the President of the United States to the Congress, dated September 17, 1787. The letter is signed by George Washington and is addressed to the members of the Congress. The letter is a copy of the original letter and is signed by George Washington.

4. The fourth part of the document is a letter from the President of the United States to the Congress, dated September 17, 1787. The letter is signed by George Washington and is addressed to the members of the Congress. The letter is a copy of the original letter and is signed by George Washington.

Este mismo valor se puede deducir, como comprobación, de la fórmula equivalente [10'] (ver tábl. 25), en la que

$$c_1 = b_2 \operatorname{sen} \gamma_{20} = \sqrt{\left(1 + \sqrt{\frac{5+2\sqrt{5}}{15}}\right)} : 2 \times \operatorname{sen} \gamma_{20} \times a_{20}$$

cuyo valor numérico es

$$c_1 = 0,94\ 72\ 73\ 6... \times \operatorname{sen} 60^\circ\ 58'\ 12,0'' \times a_{20} = 0,94\ 72\ 73\ 6... \times$$

$$\times 0,87\ 43\ 65\ 7... \times a_{20} = 0,82\ 82\ 63\ 5... \times a_{20}$$

valor coincidente con el ya obtenido anteriormente, y por lo tanto:

$$c_1 = \sqrt{\left(1 + \sqrt{\frac{5+2\sqrt{5}}{15}}\right)} : 2 \times \sqrt{\left[2 - 2\sqrt{\frac{5+2\sqrt{5}}{15}}\right]} : \left[4 \times \left(\frac{15+\sqrt{5}}{10} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}\right)\right] \times a_{20}$$

### Área lateral "S" del poliedro derivado

Se obtiene como suma de las áreas laterales de las veinte pirámides rectas de base triangular regular de lado " $b_{20}$ ", cuyas caras laterales son triángulos isósceles de base " $b_{20}$ " y altura " $p$ ", valores ya determinados.

$$S = 20 \times 3 \times \frac{b_{20} \times p}{2} = 30 \times \sqrt{\frac{10-2\sqrt{5}}{5}} \times \sqrt{\frac{15+\sqrt{5}}{10} - 2\sqrt{\frac{5+2\sqrt{5}}{15}}} \times (a_{20})^2$$

$$= 30 \times 1,05\ 14\ 62\ 2... \times 0,36\ 64\ 66\ 7... (a_{20})^2 = 11,55\ 97\ 76\ 5... (a_{20})^2$$

	<p>THE UNIVERSITY OF CHICAGO</p>	
<p>1. The first part of the paper is devoted to a discussion of the general theory of the problem. It is shown that the problem is equivalent to a certain boundary value problem for a system of partial differential equations.</p>	<p>2. In the second part of the paper, the general theory is applied to the case of a certain specific problem. It is shown that the problem is solvable and that the solution is unique.</p>	<p>3. The third part of the paper is devoted to a discussion of the numerical solution of the problem. It is shown that the problem can be solved numerically by a certain method.</p>
<p>4. The fourth part of the paper is devoted to a discussion of the physical interpretation of the results. It is shown that the results have a certain physical interpretation.</p>	<p>5. The fifth part of the paper is devoted to a discussion of the conclusions. It is shown that the results are in agreement with the physical interpretation.</p>	<p>6. The sixth part of the paper is devoted to a discussion of the references. It is shown that the results are in agreement with the references.</p>



Volumen "V" del poliedro derivado

Se obtiene como suma del volumen del icosaedro dado  $\rho$  de las veinte pirámides formadas en sus caras.

$$V = V_{20} + 20 \times \frac{S_3 \times h}{3}$$

siendo " $S_3$ " el área de una cara del icosaedro, y " $h$ " la altura de la pirámide.

Para obtener  $V_{20}$  en función de  $a_{20}$ , ver lám. 5, fórmulas 55 y 43 que nos dan

$$V_{20} = \frac{15 + 5\sqrt{5}}{12} (l_{20})^3 = \frac{15 + 5\sqrt{5}}{12} \times \left( \sqrt{\frac{10 - 2\sqrt{5}}{5}} a_{20} \right)^3 = \frac{\sqrt{8(5 + \sqrt{5})}}{3} (a_{20})^3$$

Desarrollo del cálculo anterior: 
$$\boxed{V_{20}} = \frac{15 + 5\sqrt{5}}{12} \times \left( \sqrt{\frac{10 - 2\sqrt{5}}{5}} \right)^3 (a_{20})^3 =$$

$$= \frac{15 + 5\sqrt{5}}{12} \times \frac{10 - 2\sqrt{5}}{5} \times \sqrt{\frac{2(5 - \sqrt{5})}{5}} (a_{20})^3 = \frac{(15 + 5\sqrt{5})(10 - 2\sqrt{5})}{5 \times 12} \times \sqrt{\frac{2(5 - \sqrt{5})}{5}} (a_{20})^3$$

$$= \frac{(3 + \sqrt{5})(5 - \sqrt{5})}{6} \times \sqrt{\frac{2(5 - \sqrt{5})}{5}} (a_{20})^3 = \frac{15 + 5\sqrt{5} - 3\sqrt{5} - 5}{6} \sqrt{\frac{2(5 - \sqrt{5})}{5}} (a_{20})^3 =$$

$$= \frac{10 + 2\sqrt{5}}{6} \sqrt{\frac{2(5 - \sqrt{5})}{5}} (a_{20})^3 = \frac{5 + \sqrt{5}}{3} \sqrt{\frac{2(5 - \sqrt{5})}{5}} (a_{20})^3 =$$

$$= \frac{1}{3} \sqrt{\frac{2(5 - \sqrt{5})(5 + \sqrt{5})^2}{5}} (a_{20})^3 = \frac{1}{3} \sqrt{\frac{2 \times 20 \times (5 + \sqrt{5})}{5}} (a_{20})^3 =$$

$$= \boxed{\frac{\sqrt{8(5 + \sqrt{5})}}{3} (a_{20})^3}$$

بسم الله الرحمن الرحيم  
الحمد لله رب العالمين  
والصلاة والسلام على سيدنا محمد  
النبی المصطفی  
وآله الطیبین الطاهرین  
وعلیهم السلام

اینکه در این روز  
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Igualmente "h" en función de " $a_{20}$ ", valdrá

$$h = (a_{20} - c_{20}) = \left(1 - \sqrt{\frac{5+2\sqrt{5}}{15}}\right)$$

y " $S_3$ ", también en función de " $a_{20}$ ", valdrá (ver fórmulas 43 y 54, lám. 5)

$$\begin{aligned} S_3 &= \frac{5\sqrt{3}}{20} (l_{20})^2 = \frac{5\sqrt{3}}{20} \times \left(\frac{4}{\sqrt{10+2\sqrt{5}}} a_{20}\right)^2 = \frac{\sqrt{3}}{4} \times \frac{16}{10+2\sqrt{5}} (a_{20})^2 = \\ &= \frac{2\sqrt{3}}{5+\sqrt{5}} (a_{20})^2 = \frac{2(5-\sqrt{5})\sqrt{3}}{20} (a_{20})^2 = \frac{5\sqrt{3}-\sqrt{15}}{10} (a_{20})^2 \end{aligned}$$

Substituyendo los valores de  $V_{20}$ ,  $h$  y  $S_3$  en la fórmula inicial, tendremos:

$$\begin{aligned} \boxed{V} &= V_{20} + \frac{20}{3} \times S_3 - h = \left[ \frac{\sqrt{8(5+\sqrt{5})}}{3} + \frac{20}{3} \times \frac{5\sqrt{3}-\sqrt{15}}{10} \times \left(1 - \sqrt{\frac{5+2\sqrt{5}}{15}}\right) \right] (a_{20})^3 \\ &= \boxed{\frac{10\sqrt{3}-2\sqrt{15}}{3} (a_{20})^3} \end{aligned}$$

Desarrollo del cálculo anterior:

$$\begin{aligned} \boxed{V} &= \left[ \frac{\sqrt{8(5+\sqrt{5})}}{3} + \frac{20}{3} \times \frac{5\sqrt{3}-\sqrt{15}}{10} \times \left(1 - \sqrt{\frac{5+2\sqrt{5}}{15}}\right) \right] (a_{20})^3 = \\ &= \frac{1}{3} \left[ \sqrt{8(5+\sqrt{5})} + 2(5\sqrt{3}-\sqrt{15}) \times \left(1 - \sqrt{\frac{5+2\sqrt{5}}{15}}\right) \right] (a_{20})^3 = \end{aligned}$$

$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \int_0^1 f(x) dx$

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$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \int_0^1 f(x) dx$

$$\begin{aligned}
 &= \frac{1}{3} \left[ \sqrt{8(5+\sqrt{5})} + (10\sqrt{3} - 2\sqrt{15}) - 2(5\sqrt{3} - \sqrt{15}) \sqrt{\frac{5+2\sqrt{5}}{15}} \right] (a_{20})^3 = \\
 &= \frac{1}{3} \left[ \sqrt{8(5+\sqrt{5})} + (10\sqrt{3} - 2\sqrt{15}) - 2 \sqrt{\frac{(5+2\sqrt{5})(5\sqrt{3} - \sqrt{15})^2}{15}} \right] (a_{20})^3 = \\
 &= \frac{1}{3} \left[ \sqrt{8(5+\sqrt{5})} + (10\sqrt{3} - 2\sqrt{15}) - 2 \sqrt{\frac{(5+2\sqrt{5})(75+15-10\sqrt{45})}{15}} \right] (a_{20})^3 = \\
 &= \frac{1}{3} \left[ \sqrt{8(5+\sqrt{5})} + (10\sqrt{3} - 2\sqrt{15}) - 2 \sqrt{\frac{(5+2\sqrt{5})(90-30\sqrt{5})}{15}} \right] (a_{20})^3 = \\
 &= \frac{1}{3} \left[ \sqrt{8(5+\sqrt{5})} + (10\sqrt{3} - 2\sqrt{15}) - 2 \sqrt{(5+2\sqrt{5})(6-2\sqrt{5})} \right] (a_{20})^3 = \\
 &= \frac{1}{3} \left[ \sqrt{8(5+\sqrt{5})} + (10\sqrt{3} - 2\sqrt{15}) - \sqrt{8(5+2\sqrt{5})(3-\sqrt{5})} \right] (a_{20})^3 = \\
 &= \frac{1}{3} \left[ \sqrt{8(5+\sqrt{5})} + (10\sqrt{3} - 2\sqrt{15}) - \sqrt{8(15+6\sqrt{5}-5\sqrt{5}-10)} \right] (a_{20})^3 = \\
 &= \frac{1}{3} \left[ \sqrt{8(5+\sqrt{5})} + (10\sqrt{3} - 2\sqrt{15}) - \sqrt{8(5+\sqrt{5})} \right] (a_{20})^3 = \\
 &= \boxed{\frac{10\sqrt{3} - 2\sqrt{15}}{3} (a_{20})^3}
 \end{aligned}$$



1. The first part of the document is a list of names and addresses. The names are written in a cursive script, and the addresses are written in a more formal, printed script. The list is organized into two columns, with names on the left and addresses on the right. The names are: [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible]. The addresses are: [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible].

[illegible]

En el cuadro sinóptico que damos a continuación, resumimos los resultados anteriores:

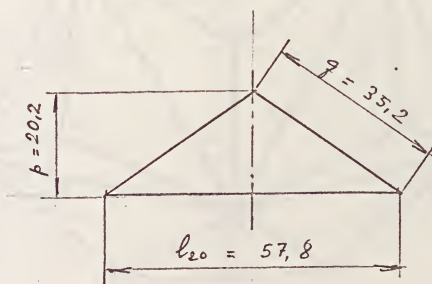
Magnitud	Valor exacto	Valor decimal aproximado
$l_{20}$	$\sqrt{\frac{10-2\sqrt{5}}{5}} a_{20}$	1, 05 14 62... $a_{20}$
$b_1$	$\sqrt{\frac{5+\sqrt{5}}{10}} a_{20}$	0, 85 06 51... $a_{20}$
$b_2$	$\sqrt{\left(1+\sqrt{\frac{5+2\sqrt{5}}{15}}\right):2} a_{20}$	0, 94 72 74... $a_{20}$
$c_{20}$	$\sqrt{\frac{5+2\sqrt{5}}{15}} a_{20}$	0, 79 46 55... $a_{20}$
$C_1$	$\frac{\sqrt{\frac{5+\sqrt{5}}{10}} \times \left[\sqrt{6(5-\sqrt{5})+2\sqrt{5}}:2\sqrt{1+\left(\frac{\sqrt{6(5-\sqrt{5})+2\sqrt{5}}}{2}\right)^2}\right]}{\sqrt{\left(1+\sqrt{\frac{5+2\sqrt{5}}{15}}\right):2} \times \left[\sqrt{2-2\sqrt{\frac{5+2\sqrt{5}}{15}}}\right]:\left[4\left(\frac{15+\sqrt{5}}{10}-2\sqrt{\frac{5+2\sqrt{5}}{15}}\right)\right]} a_{20}$	0, 82 82 64... $a_{20}$
$d_{20}$	$\sqrt{\frac{10-2\sqrt{5}}{15}} a_{20}$	0, 60 70 62... $a_{20}$
$k_{20}$	$\sqrt{\frac{5-\sqrt{5}}{30}} a_{20}$	0, 30 35 31... $a_{20}$
$2\varphi_{20}$	$\text{sen } \varphi_{20} = \frac{\sqrt{5} + \sqrt{3}}{6} \quad \varphi_{20} = 69^\circ 5' 41,4''$	$\text{sen } \varphi_{20} = 0,93 41 72...2 \varphi_{20} = 138^\circ 11' 22,8''$
$2\alpha_{20}$	$\frac{1}{2} \alpha_{20} = -\frac{\sqrt{6(5-\sqrt{5})} + 2\sqrt{5}}{2} \quad \alpha_{20} = 103^\circ 10' 26,7''$	$\frac{1}{2} \alpha_{20} = -4,27 22 16...2 \alpha_{20} = 206^\circ 20' 53,4''$
$2\gamma_{20}$	$\text{sen } \gamma_{20} = \left[\sqrt{2-2\sqrt{\frac{5+2\sqrt{5}}{15}}}\right]:\left[4\left(\frac{15+\sqrt{5}}{10}-2\sqrt{\frac{5+2\sqrt{5}}{15}}\right)\right]$	$\text{sen } \gamma_{20} = 0,87 43 66...2 \gamma_{20} = 121^\circ 56' 24,0''$
$\beta_{20}$	$\text{sen } \beta_{20} = \left(1-\sqrt{\frac{5+2\sqrt{5}}{15}}\right):\left[\sqrt{\frac{15+\sqrt{5}}{10}}-2\sqrt{\frac{5+2\sqrt{5}}{15}}\right]$	$\text{sen } \beta_{20} = 0,56 03 39...- \beta_{20} = 69^\circ 5' 41,4''$
$p$	$\sqrt{\frac{15+\sqrt{5}}{10}} - 2\sqrt{\frac{5+2\sqrt{5}}{15}} a_{20}$	0, 36 64 68... $a_{20}$
$q$	$\sqrt{2-2\sqrt{\frac{5+2\sqrt{5}}{15}}} a_{20}$	0, 64 08 52... $a_{20}$
$t$	$\sqrt{\frac{10-2\sqrt{5}}{5}} a_{20}$	1, 05 14 62... $a_{20}$
$S$	$30\sqrt{\frac{10-2\sqrt{5}}{5}} \times \sqrt{\frac{15+\sqrt{5}}{10}} - 2\sqrt{\frac{5+2\sqrt{5}}{15}} (a_{20})^2$	11, 55 97 77... $(a_{20})^2$
$V$	$\frac{10\sqrt{3}-2\sqrt{15}}{3} (a_{20})^3$	3, 19 15 14... $(a_{20})^3$

Date	Description	Amount

FIGURA CORPÓREA

Se obtiene por acoplamiento de 60 caras iguales en forma de triángulo isósceles, de base  $l_{20} = 57,8$  mm y altura  $p = 20,2$  mm; en este triángulo, el lado "q" tiene el valor  $q = 35,2$  mm (comprobación).

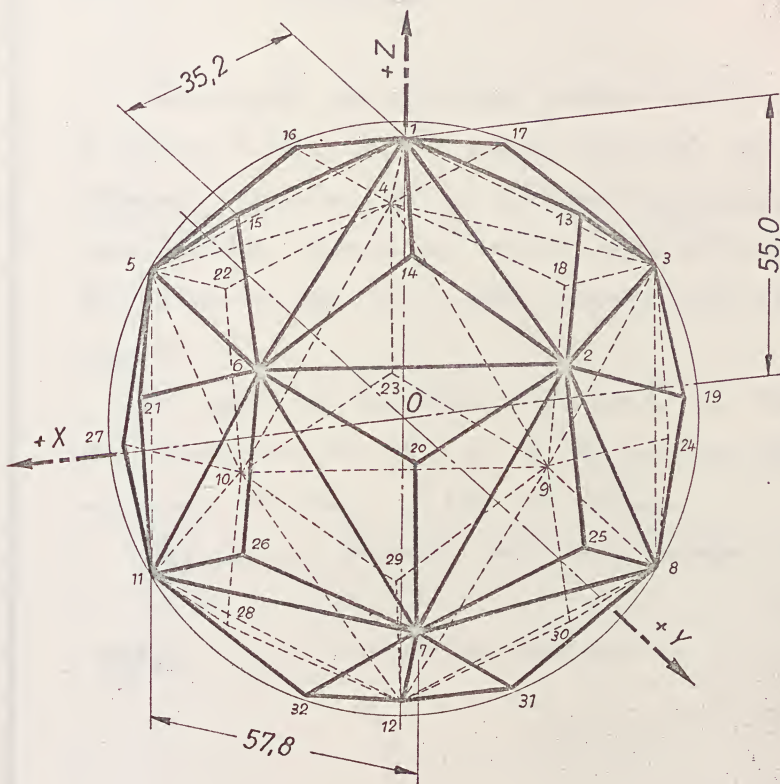
Para obtener este poliedro se formarán previamente 20 pirámides rectas de base triangular regular de lado  $l_{20} = 57,8$  mm, cuyas caras laterales son 3 triángulos (ver figura), acoplados por su lado "q"



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*Poliedro derivado del icosaedro regular*



Diagram illustrating the structure of a dodecahedron or icosahedron.

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro derivado de un tetraedro regular y de su tetraedro conjugado por sus aristas, cuando se unen consecutivamente los extremos de dos aristas correspondientes en ambos.

El radio de la esfera circunscrita al tetraedro dado, es de 55 mm, y las coordenadas de su centro O son: O (72, 72, 85) mm.

Dibujar en formato A3V y a escala 1:1

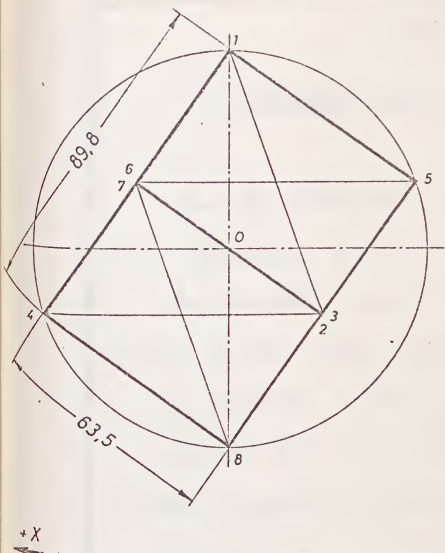
DATOS

O (72, 72, 85) mm

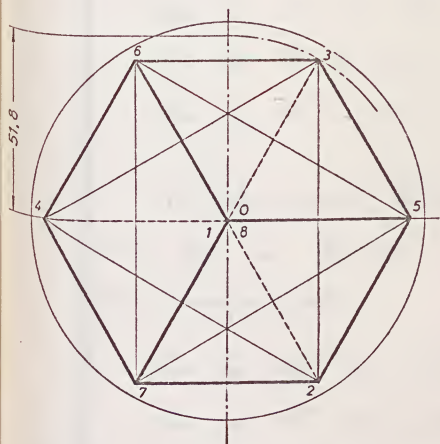
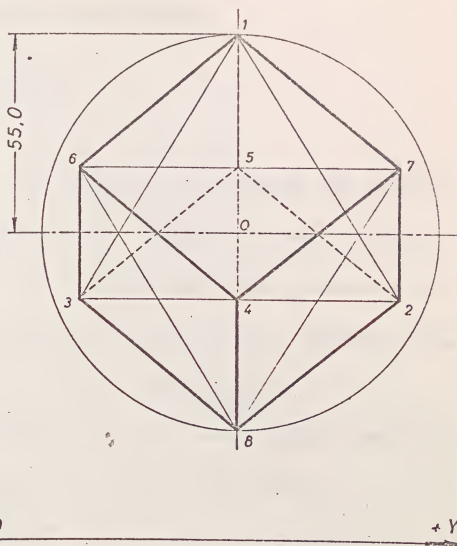
$a_4 = 55$  mm.



I



III



#### NUMERACIÓN DE VÉRTICES

Tetraedro dado (rojo)..... 1 al 4

Tetraedro conjugado (azul).... 5 al 8

Poliedro derivado (negro)..... 1 al 8

#### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro derivado de un tetraedro regular y de su tetraedro conjugado por sus aristas cuando se unen consecutivamente los extremos de dos aristas correspondientes en ambas.

El radio de la esfera circunscrita al tetraedro dado, es de 55mm, y las coordenadas de su centro O, son: O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

	Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:						Curso
Alumno:						
Escala	Derivado de los conjugados tetraedro-tetraedro					Lámina 30
1:1						Curso 19 - 73

II





## TETRAEDRO - TETRAEDRO

CONSIDERACIONES PREVIAS

En las láminas 11 a 18 hemos estudiado los poliedros regulares convexos conjugados obtenidos al trazar por los puntos medios de las aristas de un poliedro regular dado, rectas perpendiculares al plano determinado por dichas aristas y el centro de aquél.

En la lámina 11<sup>a</sup> obtuvimos el conjugado del tetraedro regular convexo, que es otro tetraedro.

En la 13<sup>a</sup> el del exaedro, que es un octaedro.

En la 14<sup>a</sup> el del octaedro, que es un exaedro.

En la 16<sup>a</sup> el del dodecaedro, que es un icosaedro, y

En la 17<sup>a</sup> el del icosaedro, que es un dodecaedro.

Por otra parte, hemos representado los sólidos comunes que se forman por la intersección de dos poliedros conjugados.

En la 12<sup>a</sup> de tetraedro y tetraedro

En la 15<sup>a</sup> de exaedro y octaedro, y

En la 18<sup>a</sup> de dodecaedro e icosaedro

En la representación de estas tres últimas láminas, podemos observar que cada dos aristas correspondientes de ambos poliedros conjugados, son coplana-



rias y se cortan perpendicularmente en sus puntos medios; por consiguiente al unir sucesivamente los extremos de dichas aristas obtenemos un cuadrilátero plano, cuyas diagonales son las mencionadas aristas (una de cada poliedro) y que tendrá sus lados iguales, por lo que será o un rombo si las dos aristas son desiguales, o un cuadrado si son iguales.

En ambos casos, y repitiendo esta operación en todas las aristas correspondientes, obtendremos un poliedro de caras cuadradas o cúbicas, todas iguales.

Éstos son los que vamos a estudiar seguidamente.

El número de poliedros diferentes, generados de esta forma, será de tres solamente y se obtendrán de los conjugados

- a) Tetraedro - tetraedro
- b) Hexaedro - octaedro
- c) Dodecaedro - icosaedro.

Pasemos a estudiar, como ejercicio de esta lámina, el primer caso a).

The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is of great importance in the theory of the structure of the atom. The second part is devoted to a detailed discussion of the problem. It is shown that the problem is of great importance in the theory of the structure of the atom. The third part is devoted to a detailed discussion of the problem. It is shown that the problem is of great importance in the theory of the structure of the atom.

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PROCESO GRÁFICO

El trazado gráfico del poliedro buscado, consiste en determinar previamente los vértices del tetraedro dado, y seguidamente los de su conjugado.

A continuación bastará unir consecutivamente, formando un cuadrilátero, los extremos de cada dos aristas perpendiculares (una de cada poliedro); estudiando en cada proyección la visibilidad de cada arista del poliedro pedido, se obtendrá la representación buscada.

Para el trazado de los vértices del tetraedro dado y de los de su conjugado, se seguirá el proceso análogo estudiado en el ejercicio de la lámina 11, por lo que omitimos su repetición.

El poliedro buscado tiene las propiedades siguientes:

1° Por ser las aristas del tetraedro dado y las de su conjugado, de igual magnitud (ambos inscritos en la misma esfera), la figura geométrica de sus caras serán cuadrados (por ser sus diagonales iguales y perpendiculares), y todas iguales.

2° Como cada cara del poliedro buscado contiene a una arista del poliedro dado. (y también

[illegible]

[illegible text block]

[illegible text block]

otra de su conjugado), el número de caras del mismo será igual al número de aristas del tetraedro; por consiguiente tendrá 6 caras

3° El número de sus vértices será pues la suma de los del tetraedro dado y los de su conjugado, teniendo por consiguiente  $4 + 4 = 8$  vértices

4° Como cada cara tiene 4 lados iguales, el número de aristas del poliedro buscado será  $\frac{1}{2} (4 \times 6) = 12$  aristas, siendo 6 el número de caras; todas las aristas serán iguales.

5° El poliedro pedido será convexo, ya que queda en un mismo semiplano al prolongar el plano de cualquier cara de aquél.

Las propiedades anteriores definen al poliedro pedido por las condiciones

$$\left. \begin{array}{l} C = 6 \\ V = 8 \\ A = 12 \\ \text{Caras cuadradas} \end{array} \right\} \text{Que son las que cumplen únicamente el } \underline{\text{Hexaedro regular}}.$$

Por consiguiente, el poliedro buscado es un cubo inscrito en la misma esfera dada.



PROCESO GRÁFICO - ANALÍTICO

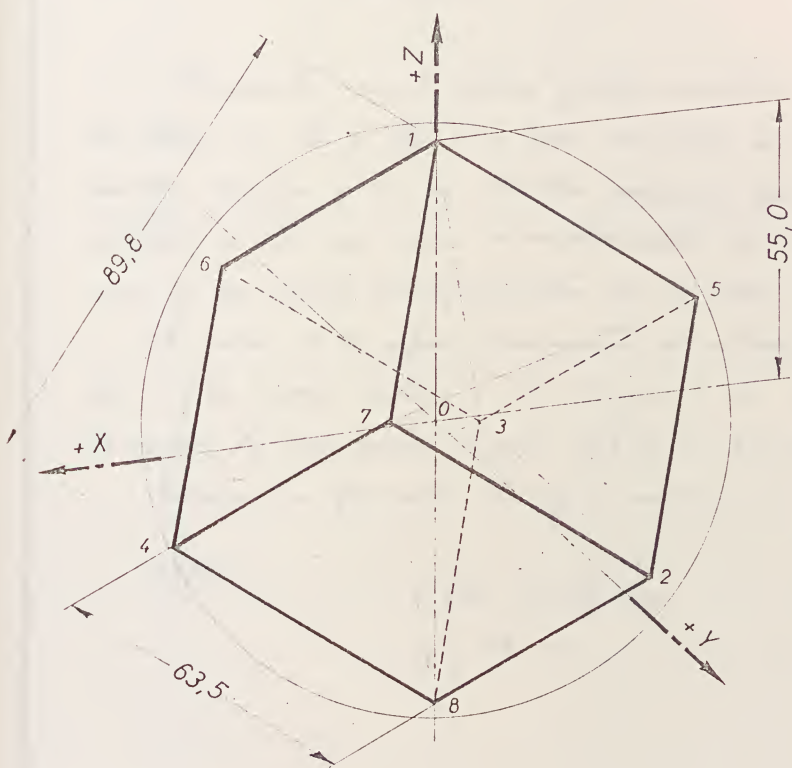
El cálculo analítico de las magnitudes acotadas, es igual al desarrollado en la lámina 25, cuyo proceso y cuadro sinóptico correspondiente es idéntico y por lo cual omitimos su repetición.

FIGURA CORPÓREA

Se obtiene por acoplamiento de seis cuadrados de 63,5 mm. de lado (ver lám. 2).







Derivado de los conjugados tetraedro-tetraedro



## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro derivado de un octaedro regular y de su octaedro conjugado por sus aristas, cuando se unen consecutivamente los extremos de dos aristas correspondientes en ambos.

El radio de la esfera circunscrita al octaedro dado (de mayor radio), es de 55 mm, y las coordenadas de su centro O son: O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

DATOS

$$O (72, 72, 85) \text{ mm}$$

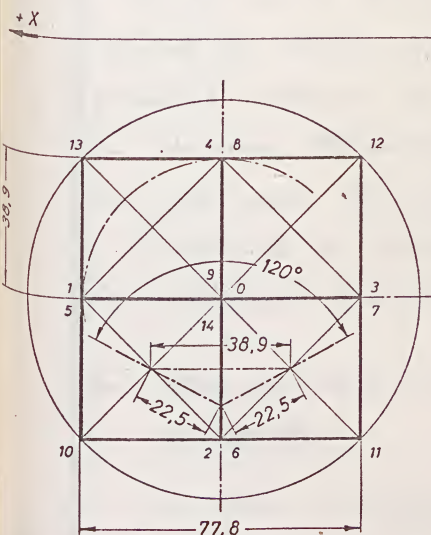
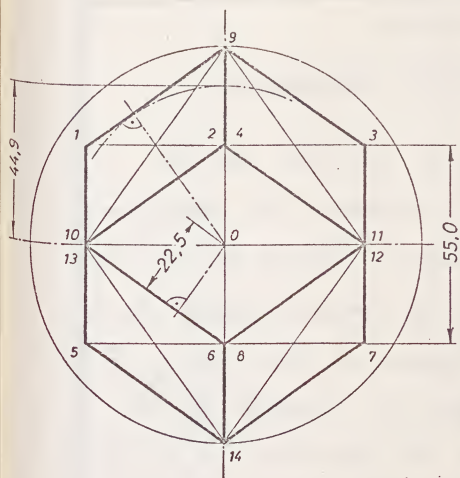
$$\alpha_0 = 55 \text{ mm}$$

The following is a list of the  
 names of the persons who have  
 been named in the report of the  
 committee on the subject of the  
 proposed amendment to the  
 constitution of the State of  
 New York, as passed by the  
 Senate on the 10th day of  
 March, 1894.

J. B. CROSSLAND  
 J. B. CROSSLAND



I

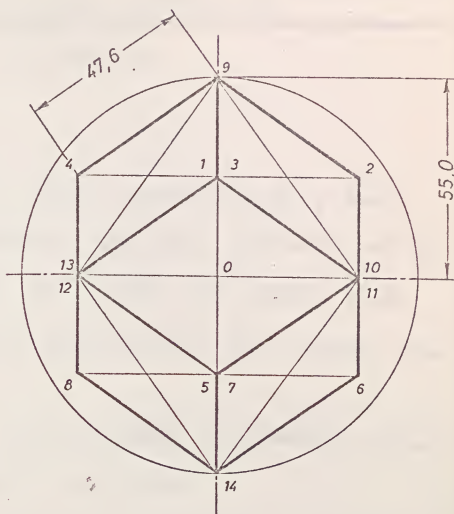


### NUMERACIÓN DE VÉRTICES

Exaedro conjugado (rojo)--- 1 al 8  
 Octaedro dado (azul)----- 9 al 14  
 Poliedro derivado----- 1 al 14

+Z

III



### ENUNCIADO

Representar por el método gráfico-analítico en los planos I, II y III, el poliedro derivado de un exaedro regular y de su octaedro conjugado por sus aristas, cuando se unen consecutivamente los extremos de dos aristas correspondientes en ambos.

El radio de la esfera circunscrita al octaedro dado es de 55 mm, y las coordenadas de su centro O, son:  
 O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

+Y

	Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:						Curso
Alumno:						
Escala	Derivado de los conjugados exaedro-octaedro					Lámina 31
1:1						Curso 19 -15

II



The first diagram is a cube with internal lines forming a star-like pattern. The second diagram is a cube with internal lines forming a star-like pattern. The third diagram is a cube with internal lines forming a star-like pattern. The fourth diagram is a cube with internal lines forming a star-like pattern. The fifth diagram is a cube with internal lines forming a star-like pattern. The sixth diagram is a cube with internal lines forming a star-like pattern. The seventh diagram is a cube with internal lines forming a star-like pattern. The eighth diagram is a cube with internal lines forming a star-like pattern. The ninth diagram is a cube with internal lines forming a star-like pattern. The tenth diagram is a cube with internal lines forming a star-like pattern.



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1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

## EXAEDRO - OCTAEDRO

CONSIDERACIONES PREVIAS

En las láminas 13 y 14 hemos estudiado los poliedros conjugados del exaedro y octaedro regulares, obtenidos al trazar por los puntos medios de las aristas del poliedro dado, rectas perpendiculares al plano determinado por dichas aristas y el centro de aquél.

En la lámina 15 hemos representado el poliedro obtenido por la intersección de ambos conjugados.

En la presente lámina 31 vamos a estudiar el poliedro derivado de ambos conjugados cuando se unen sucesivamente los extremos de cada dos aristas correspondientes con lo cual obtendremos combos todos iguales, que serán las caras del poliedro pedido.

Puramente al estudio de su trazado, vamos a deducir las propiedades geométricas del poliedro derivado.

1ª Todas sus caras son iguales y tienen la forma de combos.

En efecto, en los trazados gráficos de las láminas 13 a 15 puede observarse que las aristas del octaedro son mayores que las del exaedro. Esto puede comprobarse analíticamente mediante la fórmula 11, lám. 2 y fórmula 21, lám. 3, siguientes:

$$a_6 = \frac{\sqrt{3}}{2} l_6 \quad \text{y} \quad a_8 = \frac{\sqrt{2}}{2} l_8, \quad \text{en las que}$$



despejando  $l_6$  y  $l_8$ , tendremos

$$l_6 = \frac{2}{\sqrt{3}} a_6 = \frac{2\sqrt{3}}{3} a_6 \quad \text{y} \quad l_8 = \frac{2}{\sqrt{2}} a_8 = \sqrt{2} a_8$$

en los que haciendo  $a_6 = a_8$ , y siendo

$$\sqrt{2} > \frac{2\sqrt{3}}{3},$$

se verificará que a igualdad de radios de sus caras circunscritas, será

$$l_8 > l_6$$

y con mayor motivo, cuando como en el caso que nos ocupa, es

$$a_8 > a_6$$

Así pues al ser  $l_8 > l_6$ , el cuadrilátero obtenido al unir sucesivamente los extremos de dos aristas correspondientes en los dos poliedros conjugados, con combos, todos iguales y caras del poliedro derivado

2.º El número de caras del poliedro derivado, será de 12

En efecto, en virtud de su generación, cada cara contiene una arista del exaedro y también otra del octaedro; en ambos poliedros es de 12 el número de sus aristas.



1. The first part of the paper is devoted to a general discussion of the problem.

2. The second part is devoted to a detailed analysis of the various cases.

3. The third part is devoted to a discussion of the results obtained.

4. The fourth part is devoted to a discussion of the conclusions.

5. The fifth part is devoted to a discussion of the future work.

6. The sixth part is devoted to a discussion of the references.

7. The seventh part is devoted to a discussion of the appendix.

8. The eighth part is devoted to a discussion of the bibliography.

9. The ninth part is devoted to a discussion of the index.

10. The tenth part is devoted to a discussion of the conclusion.

3° El número de vértices del poliedro derivado es de 14

Este número será el de la suma de los vértices del exaedro (8) y del octaedro (6), generadores del poliedro.

4° El poliedro derivado es convexo

Pues al prolongar el plano de cualquiera de sus caras, queda todo él en el mismo semiespacio.

5° El número de aristas será de 24

Para ser convexo, se verificará la relación de Euler en la que

$$C + V = A + 2$$

de la que se deduce A, ya que conocemos  $C = 12$  y  $V = 14$ . Todas las aristas son iguales.

6° Los ángulos sólidos formados en los vértices del exaedro son triédros, y los formados en los vértices del octaedro, son tetraédros.

ya que en dichos vértices concurren respectivamente tres o cuatro aristas de los poliedros conjugados. Así pues existirán en el poliedro derivado 8 ángulos sólidos triédros y 6 tetraédros.

7° Existe una esfera que pasa por los vértices tetraédros



la circunscrita al octaedro dado.

8.º Existe una esfera, concéntrica con la anterior y distinta de ésta, que pasa por los vértices triédros.

la circunscrita al exaedro conjugado y de menor radio que la anterior.

9.º Existe una esfera, concéntrica con las anteriores, tangente a las aristas, no en su punto medio.

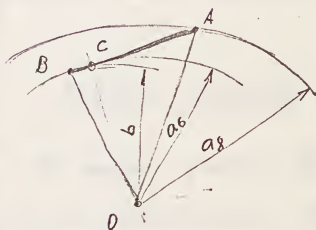


Figura 1

Si consideramos una arista cualquiera  $A-B$ , (fig. 1), cuyo extremo  $A$  pertenezca a un vértice del octaedro y el  $B$  a otro del exaedro, y unimos estos extremos con el centro  $O$  de

ambos poliedros conjugados, el plano  $OAB$  será diametral, y el

triángulo  $DAB$  tiene  $OA > OB$  por ser  $OA$  el radio  $a_8$  de la esfera circunscrita al octaedro y  $OB$  el radio  $a_6$  de la del exaedro y ya hemos visto en la propiedad 1.º que  $a_8 > a_6$ . Así pues, la altura  $OC$  de este triángulo (constante para todas las aristas), será el radio " $b$ " de la esfera tangente a dichas aristas y el pie  $C$  de dicha altura no equidistará de  $A$  y  $B$ .

10.º Existe una esfera, concéntrica con las anteriores,

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tangente a todas las caras del poliedro derivado, en el centro del rombo (esfera inscrita).

Por su el centro del rombo el punto de intersección <sup>y medio</sup> de dos aristas de los poliedros regulares conjugados (una de cada uno de ellos), dicho punto coincide con el de tangencia de la esfera común tangente a ambos poliedros, y por lo tanto, el radio que pasa por él será perpendicular a ambas aristas; por consiguiente será tangente a la cara formada por ellas.

#### PROCESO GRÁFICO

El trazado gráfico del poliedro pedido, consiste en determinar previamente los vértices del octaedro dado, y seguidamente los del octaedro conjugado.

A continuación bastará unir consecutivamente, formando un cuadrilátero (paralelogramo en las proyecciones), los extremos de cada dos aristas perpendiculares correspondientes (una de cada poliedro); estudiando en cada proyección la visibilidad de las aristas del poliedro buscado, se obtendrá fácilmente la representación de éste.

Para el trazado del octaedro dado, y del octaedro conjugado, se seguirá el mismo proceso que el estudiado en el ejercicio de la lámina 15, por lo que omitiremos su repetición.

The first thing I noticed when I stepped out of the car was the cold. It was a sharp contrast to the warm blanket of the car. I shivered slightly, but then I remembered that this was just the beginning. The air was crisp and clean, a welcome change from the stuffy interior of the car. I took a deep breath, feeling the cool air fill my lungs. The sun was shining brightly, casting a warm glow over everything. I smiled, feeling a sense of peace and tranquility. The world was so beautiful, and I was so lucky to be here. I took another deep breath, feeling the cool air fill my lungs. The sun was shining brightly, casting a warm glow over everything. I smiled, feeling a sense of peace and tranquility. The world was so beautiful, and I was so lucky to be here.

The second thing I noticed was the smell. It was a mix of fresh air and the scent of the flowers that were in bloom. I closed my eyes and breathed in deeply, feeling the fragrance fill my nostrils. The smell was so comforting, so familiar. It reminded me of home, of the garden that I had planted with my own hands. I smiled, feeling a sense of peace and tranquility. The world was so beautiful, and I was so lucky to be here. I took another deep breath, feeling the cool air fill my lungs. The sun was shining brightly, casting a warm glow over everything. I smiled, feeling a sense of peace and tranquility. The world was so beautiful, and I was so lucky to be here.

PROCESO GRAFICO-ANALÍTICO

Para simplificar y dar más exactitud al trazado, es muy útil el empleo de cotas calculadas previamente en forma analítica.

En este ejercicio consideraremos las siguientes magnitudes del poliedro derivado:

$L$  = Arista del poliedro

$a_1$  = Radio de la esfera que pasa por los vértices tetraédricos (los del octaedro dado).

$a_2$  = Radio de la esfera que pasa por los vértices triédricos (los del escaedro conjugado)

$b$  = Radio de la esfera tangente a las aristas

$c$  = Radio de la esfera tangente a las caras

$l_8$  = Arista del octaedro dado

$l_6$  = Arista del cubo conjugado.

$l_{III}$  = Distancia entre los centros de dos caras contiguas.

$p$  = Distancia del centro de una cara a uno de sus lados

$2\varphi$  = Ángulo rectilíneo del diedro formado por dos caras contiguas.

$S$  = Superficie lateral

$V$  = Volumen

No.	Name of the person to whom the money is paid	Date
1	John Doe	1/1/1900
2	Jane Smith	2/1/1900
3	Robert Brown	3/1/1900
4	Mary White	4/1/1900
5	James Black	5/1/1900
6	Elizabeth Green	6/1/1900
7	William Red	7/1/1900
8	Sarah Blue	8/1/1900
9	Thomas Yellow	9/1/1900
10	Anna Purple	10/1/1900
11	George Grey	11/1/1900
12	Mary Black	12/1/1900
13	John White	1/1/1901



Todas las magnitudes anteriores las calcularemos en función de  $a_1$ , radio de la esfera circunscrita al octaedro regular dado.

$a_1$  = Radio de la esfera que pasa por los vértices tetraédricos.

Dato del ejercicio

$l_2$  = Arista del octaedro dado

De la fórmula 137, lám. 13, se obtiene que

$$a_1 = a_2 = l_6 \quad \text{y de la fórm. 136, que}$$

$$l'_2 = \boxed{l_2} = \sqrt{2} \, l_6 = \boxed{\sqrt{2} \, a_1}$$

$l_6$  = Arista del cubo conjugado

Pa el cálculo anterior vemos también que

$$\boxed{l_6} = \boxed{a_1}$$

$l$  = Arista del poliedro

$l_2$  y  $l_6$  son las diagonales del rombo que forma una cara, por lo que

$$\boxed{l} = \sqrt{\left(\frac{l_2}{2}\right)^2 + \left(\frac{l_6}{2}\right)^2} = \sqrt{\left(\frac{\sqrt{2} \, a_1}{2}\right)^2 + \left(\frac{a_1}{2}\right)^2} = \boxed{\frac{\sqrt{3}}{2} \, a_1}$$

Desarrollo del cálculo anterior:  $\boxed{l} = \sqrt{\left(\frac{\sqrt{2} \, a_1}{2}\right)^2 + \left(\frac{a_1}{2}\right)^2} =$



1. The first part of the question is about the definition of a function. A function is a relation between a set of inputs and a set of possible outputs, where each input is related to exactly one output.

2. The second part of the question is about the domain and range of a function. The domain is the set of all possible inputs, and the range is the set of all possible outputs.

3. The third part of the question is about the graph of a function. A graph is a visual representation of a function, showing the relationship between the input and output values.

$$f(x) = x^2 + 2x + 1$$

4. The fourth part of the question is about the properties of a function. A function is said to be one-to-one if each output is related to exactly one input. A function is said to be onto if every output is related to at least one input.

$$f(x) = x^2$$

5. The fifth part of the question is about the composition of functions. The composition of two functions  $f$  and  $g$  is a new function  $h$  defined by  $h(x) = f(g(x))$ .

$$f(x) = x^2, g(x) = x + 1, h(x) = f(g(x)) = (x + 1)^2$$

6. The sixth part of the question is about the inverse of a function. The inverse of a function  $f$  is a function  $f^{-1}$  such that  $f^{-1}(f(x)) = x$  and  $f(f^{-1}(y)) = y$ .

$$= \sqrt{\frac{2}{4} (a_1)^2 + \frac{1}{4} (a_1)^2} = \sqrt{\frac{3}{4}} (a_1) = \boxed{\frac{\sqrt{3}}{2} a_1}$$

$p$  = Distancia del centro de una cara a uno de sus lados

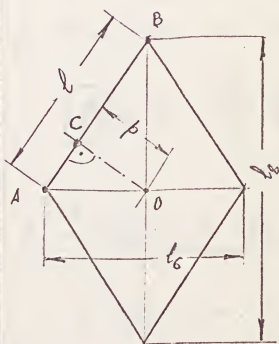


Figura 2

En la figura 2 hemos representado una cara del poliedro pedido (cambio de diagonales  $l_6$  y  $l_8$ ).

Tracemos por  $O$  la perpendicular al lado  $AB$ , siendo  $C$  el pie de la misma.

Los triángulos rectángulos  $A \cdot O \cdot B$

y  $A \cdot C \cdot O$  son semejantes (ángu-

lo  $\widehat{CAO}$  común), por lo que

$$\frac{\overline{AO}}{\overline{OC}} = \frac{\overline{AB}}{\overline{BO}} \quad \text{de donde} \quad \overline{OC} = \frac{\overline{AO} \times \overline{BO}}{\overline{AB}}$$

y sustituyendo valores  $\overline{OC} = p$ ,  $\overline{AO} = \frac{l_6}{2}$ ,  $\overline{BO} = \frac{l_8}{2}$ ,

$\overline{AB} = l$ , tendremos

$$p = \frac{\frac{l_6}{2} \times \frac{l_8}{2}}{l} = \frac{l_6 \times l_8}{4l};$$

los valores de  $l_6$ ,  $l_8$ ,  $l$ , han sido calculados anteriormente por lo que sustituyendo

$$\boxed{A} = \frac{a_1 \times \sqrt{2} \times a_1}{4 \times \frac{\sqrt{3}}{2} \times a_1} = \frac{\sqrt{2}}{2\sqrt{3}} a_1 = \boxed{\frac{\sqrt{6}}{6} a_1}$$

The first part of the problem is to find the area of the rectangle. We know that the length is 10 units and the width is 5 units. The area of a rectangle is given by the formula:

$$A = l \times w$$

where  $l$  is the length and  $w$  is the width. Substituting the given values, we get:

$$A = 10 \times 5 = 50$$

So, the area of the rectangle is 50 square units.

The second part of the problem is to find the perimeter of the rectangle. The perimeter of a rectangle is given by the formula:

$$P = 2l + 2w$$

where  $l$  is the length and  $w$  is the width. Substituting the given values, we get:

$$P = 2(10) + 2(5) = 20 + 10 = 30$$

So, the perimeter of the rectangle is 30 units.



The third part of the problem is to find the area of the triangle. We know that the base is 10 units and the height is 5 units. The area of a triangle is given by the formula:

$$A = \frac{1}{2} \times b \times h$$

where  $b$  is the base and  $h$  is the height. Substituting the given values, we get:

$$A = \frac{1}{2} \times 10 \times 5 = 25$$

So, the area of the triangle is 25 square units.

The fourth part of the problem is to find the perimeter of the triangle. The perimeter of a triangle is given by the formula:

$$P = a + b + c$$

where  $a$ ,  $b$ , and  $c$  are the lengths of the three sides. Substituting the given values, we get:

$$P = 10 + 5 + 13 = 28$$

So, the perimeter of the triangle is 28 units.

$a_2 =$  Radio de la esfera que pasa por los vértices triédros

Es el radio de la esfera, circunscrita al cubo de lado  $l_6$  dado, siendo el valor de éste, en función de  $a_1$ ,

$$l_6 = a_1$$

y teniendo en cuenta la fórm. 11, lám. 2, será

$$\boxed{a_2} = a_6 = \frac{\sqrt{3}}{2} l_6 = \boxed{\frac{\sqrt{3}}{2} a_1}$$

$b_1^* =$  Radio de la esfera tangente a las aristas

Si unimos los extremos  $C$  y  $B$  (fig. 3) de una arista del poliedro derivado, con el centro  $A$  de la esfera circunscrita, se nos formará un triángulo  $\triangle ABC$ , de

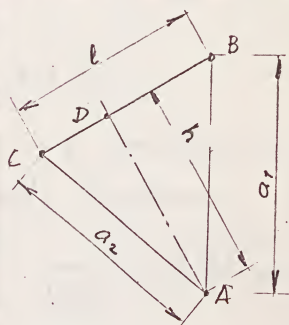


Figura 3

lados  $\overline{CB} = l$ ;  $\overline{AC} = a_2$  y  $\overline{AB} = a_1$ . La altura  $\overline{AD}$  correspondiente al lado  $\overline{CB}$ , será el radio pedido.

En Geometría se demuestra que el área  $F$  del triángulo  $ABC$ , tiene el valor

$F = \frac{1}{2} ah = \sqrt{s(s-a)(s-b')(s-c)}$ , siendo  $s$  el semiperímetro,  $\underline{a}$ ,  $\underline{b}'$   $\subseteq$  los lados del triángulo, y  $\underline{h}$  la altura correspondiente al lado  $\underline{a}$ . De aquí se

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The second of these is the fact that the  
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The third of these is the fact that the  
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deduce que

$$h = \frac{2 \sqrt{s(s-a)(s-b)(s-c)}}{a}$$

En el caso particular que nos ocupa, tendremos que

$$\left. \begin{aligned} a &= b = \frac{\sqrt{3}}{2} a_1 \\ b' &= a_2 = \frac{\sqrt{3}}{2} a_1 \\ c &= a_1 \end{aligned} \right\} \begin{aligned} s &= \frac{a+b'+c}{2} = \frac{\frac{\sqrt{3}}{2} a_1 + \frac{\sqrt{3}}{2} a_1 + a_1}{2} = \\ &= \frac{\sqrt{3}+1}{2} a_1 \end{aligned}$$

$$s-a = \frac{\sqrt{3}+1}{2} a_1 - \frac{\sqrt{3}}{2} a_1 = \frac{1}{2} a_1$$

$$s-b = \frac{\sqrt{3}+1}{2} a_1 - \frac{\sqrt{3}}{2} a_1 = \frac{1}{2} a_1$$

$$s-c = \frac{\sqrt{3}+1}{2} a_1 - a_1 = \left( \frac{\sqrt{3}+1}{2} - 1 \right) a_1 = \frac{\sqrt{3}-1}{2} a_1$$

$$h = b$$

y substituyendo estos valores en la fórmula general, tendremos

$$\boxed{b} = \frac{2 \sqrt{\frac{\sqrt{3}+1}{2} a_1 \times \frac{1}{2} a_1 \times \frac{1}{2} a_1 \times \frac{\sqrt{3}-1}{2} a_1}}{\frac{\sqrt{3}}{2} a_1} = \boxed{\frac{\sqrt{6}}{3} a_1}$$

Desarrollo del cálculo anterior:

$$\boxed{b} = \frac{2 \sqrt{\frac{\sqrt{3}+1}{2} a_1 \times \frac{1}{2} a_1 \times \frac{1}{2} a_1 \times \frac{\sqrt{3}-1}{2} a_1}}{\frac{\sqrt{3}}{2} a_1} = \frac{2 \sqrt{\frac{(\sqrt{3}+1)(\sqrt{3}-1)}{4} \times \frac{1}{4} (a_1)^2}}{\frac{\sqrt{3}}{2} a_1} =$$

$\frac{1}{2} \frac{d}{dt} \left( \frac{1}{2} m v^2 \right) = \frac{1}{2} m v \frac{dv}{dt}$

$= \frac{1}{2} m v \frac{dv}{dt}$

$$\begin{aligned}
 & \frac{1}{2} m v \frac{dv}{dt} = \frac{1}{2} m v \frac{dv}{dt} \\
 & \frac{1}{2} m v \frac{dv}{dt} = \frac{1}{2} m v \frac{dv}{dt}
 \end{aligned}$$

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$\frac{1}{2} m v \frac{dv}{dt} = \frac{1}{2} m v \frac{dv}{dt}$

NOTA. - El proceso de cálculo seguido en este estudio para la determinación del valor del radio b de la esfera tangente a las aristas, conduce con relativa facilidad a la obtención del resultado buscado.

Al tratar de aplicar este mismo proceso ~~en~~ en el ejercicio análogo de la lámina 32 para los conjugados dodecaedro-icosaedro, tropezamos con la dificultad material de simplificar un complicado radical que tenía la siguiente expresión:

$$b = \frac{\sqrt{\left(\sqrt{\frac{3-\sqrt{5}}{2}} + \sqrt{\frac{3(5-\sqrt{5})}{10}} + 1\right) \frac{a_1}{2} \times \left(\sqrt{\frac{3(5-\sqrt{5})}{10}} + 1\right) \frac{a_1}{2} \times \left(\sqrt{\frac{3-\sqrt{5}}{2}} + 1\right) \frac{a_1}{2} \times \left(\sqrt{\frac{3-\sqrt{5}}{2}} + \sqrt{\frac{3(5-\sqrt{5})}{10}}\right) \frac{a_1}{2}}{\sqrt{\frac{3-\sqrt{5}}{2}} a_1}$$

Después de varios intentos infructuosos, enfocamos de nuevo el problema bajo un punto de vista trigonométrico, ~~que~~ que nos llevó rápidamente a su solución, ~~y~~ obteniendo el valor de

$$b = \frac{2\sqrt{5}}{5} a_1$$

Este valor será pues el resultado de la simplificación del ~~el~~ complicado radical primitivo.

Si aplicamos el proceso seguido en la lámina 32, a este caso obtendremos el mismo resultado por un camino más corto que detallamos a continuación:

(véase al dorso de la  
página 10)

the first thing I did on coming to the  
city was to go to the library and see  
what was going on. I found that the  
people were very much interested in  
the new books and papers that I had  
brought with me.

As the people of the city were very  
interested in the new books and papers  
that I had brought with me, I was  
very much interested in the people of  
the city.

THE HISTORY OF THE CITY OF NEW YORK

from the first settlement of the city  
to the present time. The history of the  
city of New York is a very interesting  
one.

The history of the city of New York  
is a very interesting one. It is a  
history of the city of New York.

It is a history of the city of New York.  
It is a history of the city of New York.  
It is a history of the city of New York.

$$a_2 = \frac{\sqrt{3}}{2} a_1$$

$$l = \frac{\sqrt{3}}{2} a_1$$

$$\cos \alpha = \frac{(a_2)^2 + l^2 - (a_1)^2}{2 a_2 l} = \frac{\left(\frac{\sqrt{3}}{2} a_1\right)^2 + \left(\frac{\sqrt{3}}{2} a_1\right)^2 - (a_1)^2}{2 \times \frac{\sqrt{3}}{2} a_1 \times \frac{\sqrt{3}}{2} a_1} = \frac{\frac{3}{4} + \frac{3}{4} - 1}{\frac{2 \times 3}{4}} =$$

$$= \left(\frac{3}{2} - 1\right) : \frac{3}{2} = \frac{1}{2} : \frac{3}{2} = \frac{1}{3}$$

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2}{3} \sqrt{2}$$

y finalmente

$$\boxed{b} = a_2 \sin \alpha = \frac{\sqrt{3}}{2} a_1 \times \frac{2\sqrt{2}}{3} = \frac{\sqrt{6}}{3} a_1$$

Valor coincidente con el ya obtenido.





1870

1. The first part of the book is devoted to a general introduction to the subject of the history of the English language.

2. The second part of the book is devoted to a detailed account of the history of the English language from the time of the Anglo-Saxons to the present day.

3. The third part of the book is devoted to a detailed account of the history of the English language from the time of the Anglo-Saxons to the present day.

4. The fourth part of the book is devoted to a detailed account of the history of the English language from the time of the Anglo-Saxons to the present day.

5. The fifth part of the book is devoted to a detailed account of the history of the English language from the time of the Anglo-Saxons to the present day.

6. The sixth part of the book is devoted to a detailed account of the history of the English language from the time of the Anglo-Saxons to the present day.

$$= \left[ 2 \sqrt{\frac{2}{4} \times \frac{1}{4}} ; \frac{\sqrt{3}}{2} \right] a_1 = \left[ \frac{2\sqrt{2}}{4} ; \frac{\sqrt{3}}{2} \right] a_1 = \frac{\sqrt{2}}{\sqrt{3}} a_1 = \boxed{\frac{\sqrt{6}}{3} a_1}$$

$C =$  Radio de la esfera tangente a las caras

Es igual al radio de la esfera tangente a las aristas de uno de los dos poliedros conjugados dados.

Tomando como base el exaedro, su arista, determinada anteriormente en función de  $a_1$ , es de

$$l_6 = a_1$$

y el radio de la esfera tangente a las aristas de este cubo, valdrá (ver fórm. 12, lám. 2)

$$\boxed{C} = l_6 = \frac{\sqrt{2}}{2} l_6 = \boxed{\frac{\sqrt{2}}{2} a_1}$$

Se obtendría este mismo valor partiendo del octaedro, cuya arista

$$l_8 = \sqrt{2} a_1$$

y el radio de la esfera tangente (ver fórm. 22, lám. 3)

$$C = l_8 = \frac{1}{2} l_8 = \frac{1}{2} \sqrt{2} a_1 = \frac{\sqrt{2}}{2} a_1$$

valor coincidente con el anterior.

$l_{III} =$  Distancia entre los centros de dos caras contiguas



En el estudio que hicimos en la lámina 15 del poliedro obtenido por la intersección del exaedro y octaedro conjugado por sus aristas, vimos que el sólido común a ambos es un poliedro no regular, convexo, compuesto de 6 cuadrados y 8 triángulos equiláteros de lados de igual longitud.

Este poliedro es un caso particular de los más generales denominados "Poliedros arquimedianos" y lo hemos designado con el nombre de "Arquimediano III".

En la lámina 34 se ha efectuado la representación de dicho Arquimediano III, así como el cálculo analítico de sus principales magnitudes, que tienen aplicación a este ejercicio.

En efecto, al unir los centros de dos caras contiguas del poliedro estudiado en esta lámina, se obtiene un "Arquimediano III", cuyo lado  $l_{III}$  es la dimensión que deseamos obtener ahora, y cuya esfera circunscrita es coincidente con la esfera tangente a las aristas del exaedro y octaedro dados.

Mas como por otra parte, el radio de la esfera circunscrita al Arquimediano III es igual a su lado (ver lám. 34 (?) fórmula), tendremos finalmente

$$\boxed{l_{III}} = b = \boxed{\frac{\sqrt{2}}{2} a_1}$$





$2\varphi$  = ángulo rectilíneo del diedro formado por dos caras contiguas

Si consideramos el diedro formado por dos caras contiguas en una arista cualquiera del poliedro derivado, y cortamos dicho diedro por un plano perpendicular a la arista, que pase al mismo tiempo por el centro de una de las caras, dicho plano pasará igualmente por el centro de la otra (todas las caras son iguales); las intersecciones de dicho plano con las dos caras, serán lados del ángulo rectilíneo del diedro.

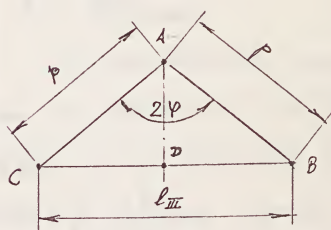


Figura 4

Si unimos seguidamente los centros de las dos caras se nos formará un triángulo isósceles A·B·C (fig. 4), cuya base C·B es la magnitud  $l_{III}$ , ya calculada, y los lados iguales la distancia del centro de la cara a la arista, o sea la magnitud  $p$

también obtenida anteriormente. El ángulo A opuesto a la base será el buscado.

Su valor será:

$$\boxed{\sin \varphi} = \frac{CD}{CA} = \frac{\frac{1}{2} l_{III}}{p} = \frac{\frac{1}{2} \times \frac{\sqrt{2}}{2} a_1}{\frac{\sqrt{6}}{6} a_1} = \boxed{\frac{\sqrt{3}}{2}}$$

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su valor numérico, será:

$$\cos \varphi = \frac{\sqrt{3}}{2} \quad \varphi = 60^\circ \quad 2\varphi = 120^\circ$$

S = Superficie lateral

El área  $S_1$  de una cara (rombo) se deduce de sus diagonales (fig. 2), y será:

$$S_1 = \frac{l_6 \times l_8}{2} = \frac{a_1 \times \sqrt{2} a_1}{2} = \frac{\sqrt{2}}{2} (a_1)^2$$

y la total  $S$  del poliedro (12 caras)

$$\boxed{S} = 12 \times \frac{\sqrt{2}}{2} (a_1)^2 = \boxed{6\sqrt{2} (a_1)^2}$$

V = Volumen

Si consideramos unidos los vértices de una cara, con el centro de la esfera circunscrita al poliedro derivado, se nos formará una pirámide recta de base cóncava y altura igual al radio  $c$  de la esfera inscrita. El volumen  $V_1$  de dicha pirámide será pues

$$V_1 = S_1 \times \frac{1}{3} c = \frac{\sqrt{2}}{2} (a_1)^2 \times \frac{\sqrt{2}}{2 \times 3} (a_1) = \frac{1}{6} (a_1)^3$$

y el volumen  $V$  total del poliedro (12 pirámides)

$$\boxed{V} = 12 \times \frac{1}{6} (a_1)^3 = \boxed{2 (a_1)^3}$$

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined on the interval  $[0, 1]$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \frac{1}{x^2}$ . The function  $f(x)$  is also shown to be concave up on the interval  $[0, 1]$ .

In the second part of the paper, the function  $f(x)$  is used to define a new function  $g(x)$  on the interval  $[0, 1]$ . The function  $g(x)$  is defined by the formula  $g(x) = f(x) + \frac{1}{x}$ . It is shown that  $g(x)$  is also continuous and differentiable on the interval  $[0, 1]$ . The derivative of  $g(x)$  is given by the formula  $g'(x) = \frac{1}{x^2} - \frac{1}{x^2} = 0$ .

The third part of the paper is devoted to the study of the properties of the function  $g(x)$ . It is shown that  $g(x)$  is a constant function on the interval  $[0, 1]$ . The value of the constant is given by the formula  $g(x) = 1$ .

The fourth part of the paper is devoted to the study of the properties of the function  $h(x)$  defined on the interval  $[0, 1]$ . The function  $h(x)$  is defined by the formula  $h(x) = f(x) + \frac{1}{x^2}$ . It is shown that  $h(x)$  is also continuous and differentiable on the interval  $[0, 1]$ . The derivative of  $h(x)$  is given by the formula  $h'(x) = \frac{1}{x^2} - \frac{2}{x^3}$ .

In the fifth part of the paper, the function  $h(x)$  is used to define a new function  $k(x)$  on the interval  $[0, 1]$ . The function  $k(x)$  is defined by the formula  $k(x) = h(x) + \frac{1}{x^3}$ . It is shown that  $k(x)$  is also continuous and differentiable on the interval  $[0, 1]$ . The derivative of  $k(x)$  is given by the formula  $k'(x) = \frac{1}{x^2} - \frac{2}{x^3} + \frac{3}{x^4}$ .

The sixth part of the paper is devoted to the study of the properties of the function  $k(x)$ . It is shown that  $k(x)$  is a constant function on the interval  $[0, 1]$ . The value of the constant is given by the formula  $k(x) = \frac{1}{2}$ .

En el cuadro sinóptico que damos a continuación resumimos los resultados anteriores.

Magnitud	Valor exacto	Valor decimal aproximado
$l$	$\frac{\sqrt{3}}{2} a_1$	0,86 60 25... $a_1$
$a_2$	$\frac{\sqrt{3}}{2} a_1$	0,86 60 25... $a_1$
$b$	$\frac{\sqrt{6}}{3} a_1$	0,81 64 97... $a_1$
$c$	$\frac{\sqrt{2}}{2} a_1$	0,70 71 07... $a_1$
$l_8$	$\sqrt{2} a_1$	1,41 42 14... $a_1$
$l_6$	$a_1$	1,00 00 00... $a_1$
$l_{III}$	$\frac{\sqrt{2}}{2} a_1$	0,70 71,07... $a_1$
$P$	$\frac{\sqrt{6}}{6} a_1$	0,40 82 48... $a_1$
$2\psi$	$\text{sen } \psi = \frac{\sqrt{3}}{2}$	$\text{sen } \psi = 0,86 60 25...$ $\psi = 60^\circ \quad 2\psi = 120^\circ$
$S$	$6\sqrt{2} (a_1)^2$	8,48 52 81... $(a_1)^2$
$V$	$2 (a_1)^3$	2,00 00 00... $(a_1)^3$
Relaciones entre magnitudes		
$l = a_2 \quad l_8 = 2c = 2 l_{III} \quad c = l_{III}$		



Page No.

Project Title: *Study on the effect of temperature on the rate of reaction of potassium permanganate with oxalic acid.*

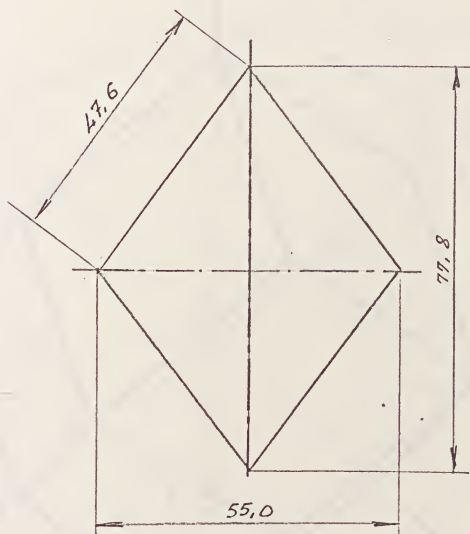
Date

The following table shows the time taken for the reaction to complete at different temperatures. The concentration of the reactants was kept constant.

Temperature (°C)	Time taken (s)	Rate of reaction (1/time)
20	120	0.0083
25	90	0.0111
30	75	0.0133
35	60	0.0167
40	45	0.0222
45	30	0.0333
50	20	0.0500
55	15	0.0667
60	10	0.1000
65	8	0.1250
70	6	0.1667
75	5	0.2000
80	4	0.2500
85	3	0.3333
90	2	0.5000
95	1	1.0000
100	1	1.0000
Average rate of reaction		
50	45	0.0222

FIGURA CORPÓREA

Se obtiene por acoplamiento de 12 conos, cuyas diagonales son los lados de los dos poliedros conjugados dados.

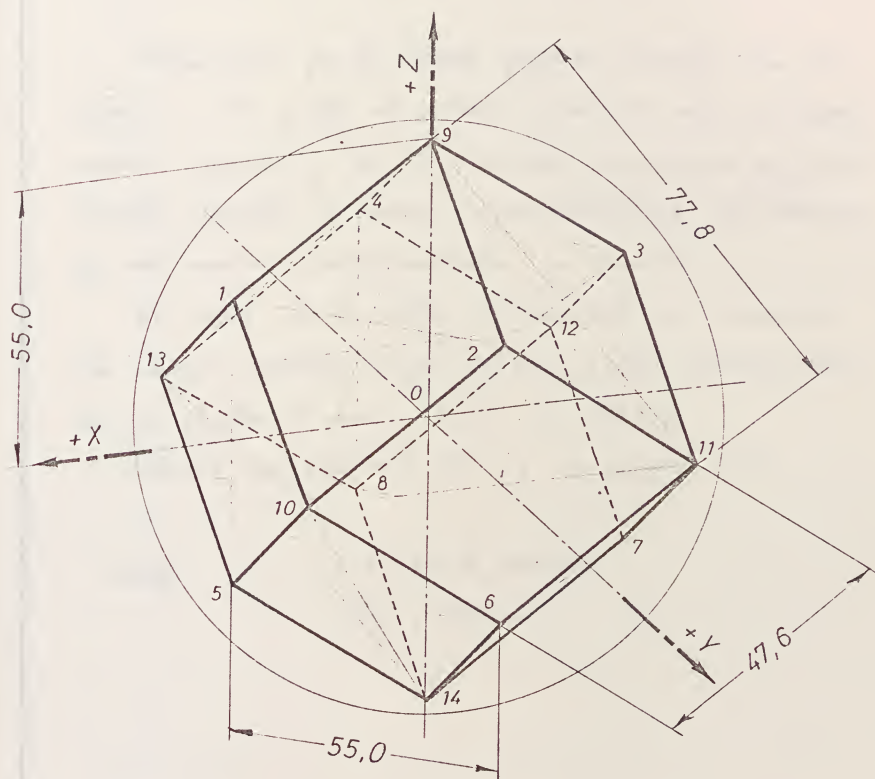


La longitud de la arista (47,6 mm) sirve de comprobación al trazado.

The first of the series of papers  
 is the first of the series of papers  
 is the first of the series of papers



The first of the series of papers  
 is the first of the series of papers



*Derivado de los conjugados exaedro-octaedro*



FIGURE 1. A 3D representation of a cube, showing the front face, the top face, and the right side face. The edges are labeled with letters: 'a' for the front edge, 'b' for the top edge, and 'c' for the right edge. The diagram is a simple line drawing with no shading.



## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro derivado de un dodecaedro regular y de su icosaedro conjugado por sus aristas, cuando se unen consecutivamente los extremos de dos aristas correspondientes en ambos.

El radio de la esfera circunscrita al icosaedro (de mayor radio), es de 55 mm, y las coordenadas de su centro O son:  $O(72, 72, 85)$  mm.

Dibujar en formato A3v y a escala 1:1.

DATOS $O(72, 72, 85)$  mm $\alpha_{20} = 55$  mm

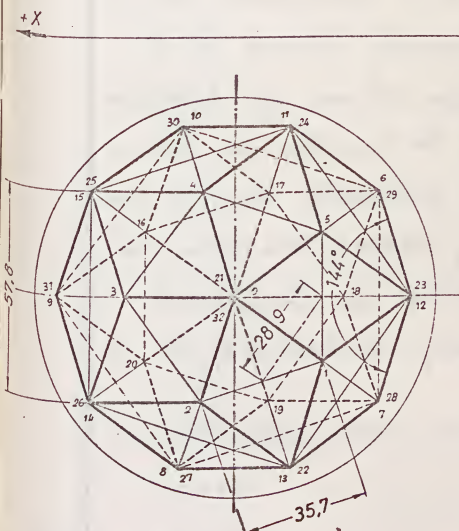
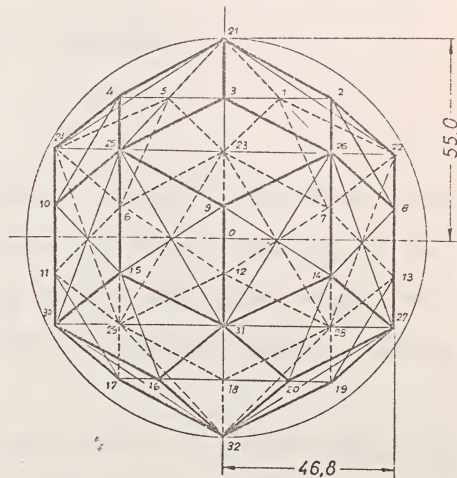
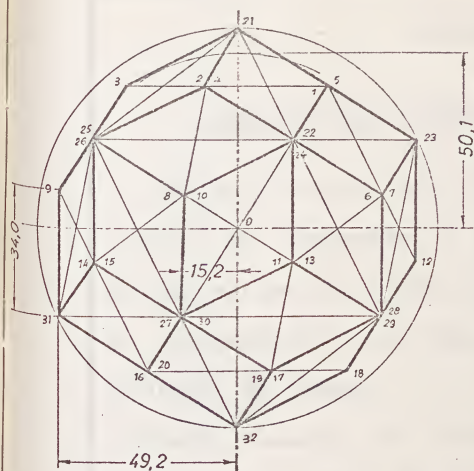
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AMERICAN MEDICAL ASSOCIATION

I

+Z

III



O

### ENUNCIADO

Representar por el método gráfico-analítico en los planos I, II y III, el poliedro derivado de un dodecaedro regular y de su icosaedro conjugado por sus aristas cuando se unen consecutivamente los extremos de dos aristas correspondientes en ambos.

El radio de la esfera circunscrita al icosaedro es de 55 mm, y las coordenadas de su centro O, son: O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

#### NUMERACIÓN DE VÉRTICES

Dodecaedro conjugado (rojo)--- 1 al 20

Icosaedro dado (azul)----- 21 al 32

Poliedro derivado (negro)----- 1 al 32

	Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:						Curso
Alumno:						
Escala	Derivado de los conjugados dodecaedro-icosaedro					Lámina 32
1:1						Curso 19 - 19

II



The first of these is the  
 cube, which is a solid  
 figure bounded by six square  
 faces, each of which is  
 a square. The cube is the  
 simplest of all the solids,  
 and is the only one which  
 can be divided into two  
 equal parts by a single  
 plane. The cube is also the  
 only solid which can be  
 divided into six equal parts  
 by three planes. The cube  
 is the only solid which can  
 be divided into eight equal  
 parts by three planes. The  
 cube is the only solid which  
 can be divided into twenty-  
 seven equal parts by three  
 planes. The cube is the only  
 solid which can be divided  
 into sixty-four equal parts  
 by three planes. The cube  
 is the only solid which can  
 be divided into one-thousand  
 equal parts by three planes.



## DODECAEDRO - ICOSAEDRO

CONSIDERACIONES PREVIAS

En las láminas 16 y 17 hemos estudiado los poliedros conjugados del dodecaedro e icosaedros regulares, obtenidos al trazar por los puntos medios de las aristas del poliedro dado, rectas perpendiculares al plano determinado por dichas aristas y el centro de aquél.

En la lámina 18 hemos representado el poliedro obtenido por la intersección de ambos conjugados.

En la presente lámina 31 vamos a estudiar el poliedro derivado de ambos conjugados cuando se unen sucesivamente los extremos de cada dos aristas correspondientes, con lo cual obtenemos rombos todos iguales, que serán las caras del poliedro pedido.

Antes de al estudio de su trazado, vamos a deducir las propiedades geométricas de este poliedro derivado.

1° Todas sus caras son iguales y tienen la forma de rombos.

En efecto, en los trazados gráficos de la las láminas 16 a 18 puede observarse que las aristas del icosaedro son mayores que las de su dodecaedro conjugado. Esto puede comprarse analíticamente mediante la fórmula 30, lám. 4, y fórmula 43, lám. 5. siguientes:



Received of \_\_\_\_\_ the sum of \_\_\_\_\_

for \_\_\_\_\_

the sum of \_\_\_\_\_ Dollars and \_\_\_\_\_ Cents  
to wit: \_\_\_\_\_ Dollars and \_\_\_\_\_ Cents  
the receipt of which is hereby acknowledged.

Witness my hand and seal this \_\_\_\_\_ day of \_\_\_\_\_

19\_\_\_\_ at \_\_\_\_\_  
\_\_\_\_\_ Secretary

\_\_\_\_\_ Treasurer

\_\_\_\_\_

\_\_\_\_\_

$$a_{12} = \frac{\sqrt{15} + \sqrt{3}}{4} l_{12} \quad \text{y} \quad a_{20} = \frac{\sqrt{10 + 2\sqrt{5}}}{4} l_{20}, \text{ en las que}$$

dejando "l<sub>12</sub>" y l<sub>20</sub>, tendremos

$$\boxed{l_{12}} = \frac{4}{\sqrt{15} + \sqrt{3}} a_{12} = \boxed{\frac{\sqrt{15} - \sqrt{3}}{3} a_{12}}$$

Desarrollo del cálculo anterior:  $\boxed{l_{12}} = \frac{4}{\sqrt{15} + \sqrt{3}} a_{12} =$

$$= \frac{4(\sqrt{15} - \sqrt{3})}{15 - 3} a_{12} = \frac{4(\sqrt{15} - \sqrt{3})}{12} a_{12} = \boxed{\frac{\sqrt{15} - \sqrt{3}}{3} a_{12}} \quad \text{y}$$

$$\boxed{l_{20}} = \frac{4}{\sqrt{10 + 2\sqrt{5}}} a_{20} = \boxed{\frac{\sqrt{10 - 2\sqrt{5}}}{5} a_{20}}$$

Desarrollo del cálculo anterior:  $\boxed{l_{20}} = \frac{4}{\sqrt{10 + 2\sqrt{5}}} a_{20} = \sqrt{\frac{16}{10 + 2\sqrt{5}}} a_{20} =$

$$= \sqrt{\frac{16(10 - 2\sqrt{5})}{100 - 20}} a_{20} = \sqrt{\frac{16(10 - 2\sqrt{5})}{80}} a_{20} = \boxed{\frac{\sqrt{10 - 2\sqrt{5}}}{5} a_{20}}$$

en las que haciendo  $a_{12} = a_{20}$ , y siendo

$$\frac{\sqrt{15} - \sqrt{3}}{2} = 0,713644... \quad \text{y} \quad \sqrt{\frac{10 - 2\sqrt{5}}{5}} = 1,051462...$$

tendremos que

$$\sqrt{\frac{10 - 2\sqrt{5}}{5}} > \frac{\sqrt{15} - \sqrt{3}}{2}$$

por lo que será

$$l_{20} > l_{12}$$

y con mayor motivo, cuando como en el caso que nos ocupa, es

$$a_{20} > a_6$$

Let  $x$  and  $y$  be real numbers such that  $x^2 + y^2 = 1$ .

Then  $x^2 \leq 1$  and  $y^2 \leq 1$ .

$$\sqrt{x^2} \leq \sqrt{1} \quad \text{and} \quad \sqrt{y^2} \leq \sqrt{1}$$

Since  $x^2 \geq 0$  and  $y^2 \geq 0$ , we have  $|x| \leq 1$  and  $|y| \leq 1$ .

$$\left| \frac{x}{y} \right| \leq \frac{|x|}{|y|} \leq \frac{1}{|y|} \leq \frac{1}{\sqrt{1-y^2}}$$

$$\left| \frac{x}{y} \right| \leq \frac{1}{\sqrt{1-y^2}}$$

Therefore,  $\left| \frac{x}{y} \right| \leq \frac{1}{\sqrt{1-y^2}}$ .

$$\left| \frac{x}{y} \right| \leq \frac{1}{\sqrt{1-y^2}}$$

Since  $|y| \leq 1$ , we have  $1-y^2 \geq 0$ .

$$\sqrt{1-y^2} \geq 0$$

$$\frac{1}{\sqrt{1-y^2}} \geq 1$$

$$\left| \frac{x}{y} \right| \leq \frac{1}{\sqrt{1-y^2}}$$

$$|x| \leq |y|$$

Thus,  $|x| \leq |y|$ .

$$|x| \leq |y|$$

Así pues, al ser  $l_{20} > l_{12}$ , el cuadrilátero obtenido al unir sucesivamente los extremos de dos aristas correspondientes en los dos poliedros conjugados, son rombos, todos iguales y caras del poliedro derivado.

2ª El número de caras del poliedro derivado, será de 30

En efecto, en virtud de su generación, cada cara contiene una arista del dodecaedro y también otra del icosaedro; en ambos poliedros es de 30 el número de sus aristas.

3ª El número de vértices del poliedro derivado es de 32

Este número será el de la suma de los vértices del dodecaedro (20) y del icosaedro (12).

4ª El poliedro derivado es convexo

Pues al prolongar el plano de cualquiera de sus caras, queda todo él en el mismo semiespacio.

5ª El número de aristas será de 60

Por ser convexo se verificará la relación de Euler, en la que

$$C + V = A + 2$$

de la que se deduce A, ya que conocemos  $C = 30$  y  $V = 32$ .  
Todas las aristas son iguales

The first of these is the fact that the  
Government has been very successful in  
the past in dealing with the various  
problems which have arisen.

It is also true that the Government has  
been very successful in dealing with the  
various problems which have arisen in  
the past. This is due to the fact that  
the Government has been very successful  
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which have arisen in the past.



6º Los ángulos sólidos formados en los vértices del dodecaedro son triedros, y los formados en los vértices del icosaedro, son pentaedros.

puesto que en dichos vértices concurren respectivamente tres o cinco aristas de los poliedros conjugados. Así pues existirán en el poliedro derivado 20 ángulos sólidos triedros y 12 pentaedros.

7º Existe una esfera que pasa por los vértices pentaedros  
la circunscrita al icosaedro dado

8º Existe una esfera, concéntrica con la anterior y distinta de ésta, que pasa por los vértices triedros.

La circunscrita al dodecaedro conjugado y de menor radio que la anterior.

9º Existe una esfera, concéntrica con las anteriores, tangente a las aristas, no en su punto medio.

Es válida la análoga demostración dada en la lámina 31.

10º Existe una esfera, concéntrica con las anteriores, tangente a todas las caras del poliedro derivado, en el centro del rombo (esfera inscrita).

La esfera común tangente a las aristas de los dos

No.	Name of the person	Age
1	John Smith	25
2	Mary Jones	30
3	Robert Brown	28
4	Elizabeth White	22
5	James Green	35
6	Sarah Black	27
7	William Grey	32

PROCESO GRÁFICO

El trazado gráfico del poliedro pedido, consiste en determinar previamente los vértices del icosaedro dado, y seguidamente los del dodecaedro conjugado.

A continuación bastará unir consecutivamente, formando un cuadrilátero (paralelogramo en las proyecciones), los extremos de cada dos aristas perpendiculares correspondientes (una de cada poliedro); estudiando en cada proyección la visibilidad de las aristas del poliedro buscado, se obtendrá fácilmente la representación de éste.

Para el trazado del icosaedro dado y del dodecaedro conjugado, se seguirá el proceso estudiado en la lámina 13, por lo que omitimos su repetición.

PROCESO GRÁFICO-ANALÍTICO

Para simplificar y dar al mismo tiempo mayor exactitud al trazado, es muy útil el empleo de cotas calculadas previamente en forma analítica.

En este ejercicio consideraremos las siguientes magnitudes del poliedro derivado:

$l$  = Arista del poliedro

$a_1$  = Radio de la esfera que pasa por los vértices pentáedricos (los del icosaedro dado)



$a_2$  = Radio de la esfera que pasa por los vértices triédros  
(los del dodecaedro conjugado)

$b$  = Radio de la esfera tangente a las aristas.

$c$  = Radio de la esfera tangente a las caras.

$l_{20}$  = Arista del icosaedro dado.

$l_{12}$  = Arista del dodecaedro conjugado.

$l_{pp}$  = Distancia entre los centros de dos caras contiguas.

$p$  = Distancia del centro de una cara a uno de sus lados.

$2\varphi$  = Ángulo rectilíneo del diedro formado por dos caras  
contiguas

$S$  = Superficie lateral

$V$  = Volumen.



Todas las magnitudes anteriores las calcularemos en  
función de  $a_1$ , radio de la esfera circunscrita al ico-  
saedro regular dado

$a_1$  = Radio de la esfera que pasa por los vértices pentáedricos

### Dato del ejercicio

$l_{20}$  = Arista del icosaedro dado

De la fórmula 165 de la lámina 16, se obtiene que

$$a_1 = a'_{20} = \frac{\sqrt{5+2\sqrt{5}}}{2} l_{12} \quad \text{y de la fórmula 164, que}$$





$$\boxed{l_{20}} = l'_{20} = \frac{1 + \sqrt{5}}{2} l_{12} = \frac{1 + \sqrt{5}}{2} \times \frac{2}{\sqrt{5 + 2\sqrt{5}}} a_1 = \boxed{\sqrt{\frac{10 - 2\sqrt{5}}{5}} a_1}$$

Desarrollo del cálculo anterior:  $\boxed{l_{20}} = \frac{1 + \sqrt{5}}{2} \times \frac{2}{\sqrt{5 + 2\sqrt{5}}} a_1 =$

$$= \frac{1 + \sqrt{5}}{\sqrt{5 + 2\sqrt{5}}} a_1 = \frac{(1 + \sqrt{5}) \sqrt{5 + 2\sqrt{5}}}{5 + 2\sqrt{5}} a_1 = \frac{(1 + \sqrt{5})(5 - 2\sqrt{5}) \sqrt{5 + 2\sqrt{5}}}{25 - 20} a_1 =$$

$$= \frac{(5 + 5\sqrt{5} - 2\sqrt{5} - 10) \sqrt{5 + 2\sqrt{5}}}{5} a_1 = \frac{(3\sqrt{5} - 5) \sqrt{5 + 2\sqrt{5}}}{5} a_1 = \frac{\sqrt{(5 + 2\sqrt{5})(3\sqrt{5} - 5)^2}}{5} a_1 =$$

$$= \frac{\sqrt{(5 + 2\sqrt{5})(45 + 25 - 30\sqrt{5})}}{5} a_1 = \frac{\sqrt{(5 + 2\sqrt{5})(70 - 30\sqrt{5})}}{5} a_1 =$$

$$= \sqrt{\frac{(5 + 2\sqrt{5})(14 - 6\sqrt{5})}{5}} a_1 = \sqrt{\frac{2(5 + 2\sqrt{5})(7 - 3\sqrt{5})}{5}} a_1 = \sqrt{\frac{2(35 + 14\sqrt{5} - 15\sqrt{5} - 30)}{5}} a_1 =$$

$$= \sqrt{\frac{2(5 - \sqrt{5})}{5}} a_1 = \boxed{\sqrt{\frac{10 - 2\sqrt{5}}{5}} a_1}$$

$l_{12}$  = Arista del dodecaedro conjugado

De la fórmula 165, lám. 16, se deduce

$$a_1 = a'_{20} = \frac{\sqrt{5 + 2\sqrt{5}}}{2} l_{12} \quad \text{de donde}$$

$$\boxed{l_{12}} = \frac{2}{\sqrt{5 + 2\sqrt{5}}} a_1 = \boxed{2 \sqrt{\frac{5 - 2\sqrt{5}}{5}} a_1}$$

Desarrollo del cálculo anterior:  $\boxed{l_{12}} = \frac{2}{\sqrt{5 + 2\sqrt{5}}} a_1 = \frac{2\sqrt{5 + 2\sqrt{5}}}{5 + 2\sqrt{5}} a_1 =$

$$= \frac{2(5 - 2\sqrt{5}) \sqrt{5 + 2\sqrt{5}}}{25 - 20} a_1 = \frac{2\sqrt{(5 + 2\sqrt{5})(5 - 2\sqrt{5})^2}}{5} a_1 = \frac{2\sqrt{(25 - 20)(5 - 2\sqrt{5})}}{5} a_1 =$$

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined on the interval  $[0, 1]$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \frac{1}{x^2}$ . The function  $f(x)$  is also shown to be concave up on the interval  $[0, 1]$ .

In the second part of the paper, the function  $f(x)$  is extended to the interval  $(1, \infty)$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \frac{1}{x^2}$ . The function  $f(x)$  is also shown to be concave up on the interval  $(1, \infty)$ .

The third part of the paper is devoted to the study of the properties of the function  $f(x)$  defined on the interval  $(-\infty, 0)$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \frac{1}{x^2}$ . The function  $f(x)$  is also shown to be concave up on the interval  $(-\infty, 0)$ .

In the fourth part of the paper, the function  $f(x)$  is extended to the interval  $(-\infty, -1)$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \frac{1}{x^2}$ . The function  $f(x)$  is also shown to be concave up on the interval  $(-\infty, -1)$ .

The fifth part of the paper is devoted to the study of the properties of the function  $f(x)$  defined on the interval  $(-1, 0)$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \frac{1}{x^2}$ . The function  $f(x)$  is also shown to be concave up on the interval  $(-1, 0)$ .

In the sixth part of the paper, the function  $f(x)$  is extended to the interval  $(0, 1)$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \frac{1}{x^2}$ . The function  $f(x)$  is also shown to be concave up on the interval  $(0, 1)$ .

The seventh part of the paper is devoted to the study of the properties of the function  $f(x)$  defined on the interval  $(1, \infty)$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \frac{1}{x^2}$ . The function  $f(x)$  is also shown to be concave up on the interval  $(1, \infty)$ .

In the eighth part of the paper, the function  $f(x)$  is extended to the interval  $(-\infty, -1)$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \frac{1}{x^2}$ . The function  $f(x)$  is also shown to be concave up on the interval  $(-\infty, -1)$ .

The ninth part of the paper is devoted to the study of the properties of the function  $f(x)$  defined on the interval  $(-1, 0)$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \frac{1}{x^2}$ . The function  $f(x)$  is also shown to be concave up on the interval  $(-1, 0)$ .

In the tenth part of the paper, the function  $f(x)$  is extended to the interval  $(0, 1)$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \frac{1}{x^2}$ . The function  $f(x)$  is also shown to be concave up on the interval  $(0, 1)$ .

$$= \frac{2\sqrt{5(5-2\sqrt{5})}}{5} a_1 = \boxed{2\sqrt{\frac{5-2\sqrt{5}}{5}} a_1}$$

$l$  = Arista del poliedro

$l_{20}$  y  $l_{12}$  son las diagonales del rombo que forma una cara, por lo que (fig. 1)

$$\begin{aligned} l &= \sqrt{\left(\frac{l_{20}}{2}\right)^2 + \left(\frac{l_{12}}{2}\right)^2} = \sqrt{\left(\frac{1}{2} \sqrt{\frac{10-2\sqrt{5}}{5}} a_1\right)^2 + \left(\frac{1}{2} \times 2 \sqrt{\frac{5-2\sqrt{5}}{5}} a_1\right)^2} = \\ &= \boxed{\sqrt{\frac{3-\sqrt{5}}{2}} a_1} \end{aligned}$$

Desarrollo del cálculo anterior:

$$\begin{aligned} l &= \sqrt{\left(\frac{1}{2} \sqrt{\frac{10-2\sqrt{5}}{5}} a_1\right)^2 + \left(\frac{1}{2} \times 2 \sqrt{\frac{5-2\sqrt{5}}{5}} a_1\right)^2} = \\ &= \sqrt{\frac{1}{4} \times \frac{10-2\sqrt{5}}{5} (a_1)^2 + \frac{5-2\sqrt{5}}{5} (a_1)^2} = \sqrt{\frac{5-\sqrt{5}}{10} + \frac{5-2\sqrt{5}}{5}} a_1 = \\ &= \sqrt{\frac{5-\sqrt{5}+10-4\sqrt{5}}{10}} a_1 = \sqrt{\frac{15-5\sqrt{5}}{10}} a_1 = \boxed{\sqrt{\frac{3-\sqrt{5}}{2}} a_1} \end{aligned}$$

$p$  = Distancia del centro de una cara a uno de sus lados

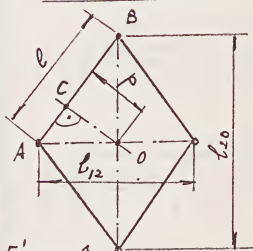


Figura 1

En la figura 1 hemos representado una cara del poliedro pedido (rombo de diagonales  $l_{20}$  y  $l_{12}$ ).

Bracemos por  $O$  la perpendicular

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tan al lado  $\overline{AB}$ , siendo  $C$  el pie de la altura.

Los triángulos rectángulos  $\overline{A.O.B}$  y  $\overline{AOC}$  son semejantes (ángulo  $\overline{C.A.O}$  común), por lo que

$$\frac{\overline{AO}}{\overline{OC}} = \frac{\overline{AB}}{\overline{BO}} \quad \text{de donde} \quad \overline{OC} = \frac{\overline{AO} \times \overline{BO}}{\overline{AB}}$$

y substituyendo los valores  $\overline{OC} = p$ ,  $\overline{AO} = \frac{l_2}{2}$ ,  $\overline{BO} = \frac{l_{20}}{2}$ ,  $\overline{AB} = l$ , tenemos:

$$p = \frac{\frac{1}{2} l_2 \times \frac{1}{2} l_{20}}{l} = \frac{l_2 \times l_{20}}{4l}; \quad \text{los valores } l_2, l_{20} \text{ y } l, \text{ son ya conocidos anteriormente}$$

por lo que

$$p = \frac{2 \sqrt{\frac{5-2\sqrt{5}}{5}} a_1 \times \sqrt{\frac{10-2\sqrt{5}}{5}} a_1}{4 \sqrt{\frac{3-\sqrt{5}}{2}} a_1} = \boxed{\sqrt{\frac{3-\sqrt{5}}{10}} a_1}$$

Desarrollo del cálculo anterior:

$$\begin{aligned} p &= \frac{2 \sqrt{\frac{5-2\sqrt{5}}{5}} a_1 \times \sqrt{\frac{10-2\sqrt{5}}{5}} a_1}{4 \sqrt{\frac{3-\sqrt{5}}{2}} a_1} = \frac{\sqrt{\frac{5-2\sqrt{5}}{5} \times \frac{10-2\sqrt{5}}{5}}}{2 \sqrt{\frac{3-\sqrt{5}}{2}}} a_1 = \\ &= \frac{1}{2} \left( \sqrt{\frac{(5-2\sqrt{5})(10-2\sqrt{5})}{25}} ; \sqrt{\frac{3-\sqrt{5}}{2}} \right) a_1 = \frac{1}{2} \times \sqrt{\frac{(5-2\sqrt{5})(10-2\sqrt{5})}{25} ; \frac{3-\sqrt{5}}{2}} \\ &= \frac{1}{2} \times \sqrt{\frac{2(5-2\sqrt{5}) \times 2(5-\sqrt{5})}{25(3-\sqrt{5})}} a_1 = \frac{1}{2} \times \frac{2}{5} \times \sqrt{\frac{(5-2\sqrt{5})(5-\sqrt{5})}{3-\sqrt{5}}} a_1 = \\ &= \frac{1}{5} \times \sqrt{\frac{25-10\sqrt{5}-5\sqrt{5}+10}{3-\sqrt{5}}} a_1 = \frac{1}{5} \sqrt{\frac{35-15\sqrt{5}}{3-\sqrt{5}}} a_1 = \frac{1}{5} \sqrt{\frac{5(7-3\sqrt{5})}{3-\sqrt{5}}} a_1 \end{aligned}$$

Q.1. \_\_\_\_\_

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\_\_\_\_\_

Q.2. \_\_\_\_\_

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Q.3. \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Q.4. \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Q.5. \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Q.6. \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Q.7. \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

$$= \frac{1}{5} \sqrt{\frac{5(7-3\sqrt{5})(3+\sqrt{5})}{4}} a_1 = \frac{1}{10} \sqrt{5(21-9\sqrt{5}+7\sqrt{5}-15)} a_1 =$$

$$= \frac{1}{10} \sqrt{5(6-2\sqrt{5})} a_1 = \frac{1}{10} \sqrt{10(3-\sqrt{5})} a_1 = \boxed{\sqrt{\frac{3(5-\sqrt{5})}{10}} a_1}$$

$a_2$  = Radio de la esfera que pasa por los vértices triedros

Es el radio de la esfera circunscrita al dodecaedro conjugado de lado  $l_{12}$ , siendo el valor de éste, en función de  $a_1$ ,

$$l_{12} = 2 \sqrt{\frac{5-2\sqrt{5}}{5}} a_1$$

y teniendo en cuenta la fórm. 30, lám. 4

$$\boxed{a_2} = a_{12} = \frac{\sqrt{15} + \sqrt{3}}{4} l_{12} = \frac{\sqrt{15} + \sqrt{3}}{4} \times 2 \sqrt{\frac{5-2\sqrt{5}}{5}} a_1 = \boxed{\sqrt{\frac{3(5-\sqrt{5})}{10}} a_1}$$

Desarrollo del cálculo anterior:  $\boxed{a_2} = \frac{\sqrt{15} + \sqrt{3}}{4} \times 2 \sqrt{\frac{5-2\sqrt{5}}{5}} a_1 =$

$$= \frac{\sqrt{15} + \sqrt{3}}{2} \times \sqrt{\frac{5-2\sqrt{5}}{5}} a_1 = \frac{1}{2} \sqrt{(\sqrt{15} + \sqrt{3})^2 \times \frac{5-2\sqrt{5}}{5}} a_1 =$$

$$= \frac{1}{2} \sqrt{(15+3+2\sqrt{45}) \times \frac{5-2\sqrt{5}}{5}} a_1 = \frac{1}{2} \sqrt{(18+6\sqrt{5}) \times \frac{5-2\sqrt{5}}{5}} a_1 =$$

$$= \frac{1}{2} \sqrt{6(3+\sqrt{5}) \times \frac{5-2\sqrt{5}}{5}} a_1 = \frac{1}{2} \sqrt{\frac{6}{5}(3+\sqrt{5})(5-2\sqrt{5})} a_1 =$$

$$= \frac{1}{2} \sqrt{\frac{6}{5}(15+5\sqrt{5}-6\sqrt{5}-10)} a_1 = \frac{1}{2} \sqrt{\frac{6}{5}(5-\sqrt{5})} a_1 = \sqrt{\frac{6(5-\sqrt{5})}{20}} a_1 =$$

$$\boxed{\sqrt{\frac{3(5-\sqrt{5})}{10}} a_1}$$

Q. 1. A particle is moving with a constant velocity of 10 m/s. Find the distance covered by it in 5 seconds.

$$s = vt$$

Where,  $s$  = distance,  $v$  = velocity,  $t$  = time

Given,  $v = 10 \text{ m/s}$ ,  $t = 5 \text{ s}$

$$s = 10 \times 5$$

$\therefore s = 50 \text{ m}$

Q. 2. A car starts from rest and accelerates uniformly to a speed of 20 m/s in 10 seconds. Find the acceleration.

$$a = \frac{v - u}{t}$$

$$a = \frac{20 - 0}{10}$$

$$a = \frac{20}{10}$$

$$a = 2 \text{ m/s}^2$$

Q. 3. A ball is thrown vertically upwards with an initial velocity of 15 m/s. Find the maximum height reached by the ball.

$$v^2 = u^2 - 2gh$$

$b$  = Radio de la esfera tangente a las aristas.

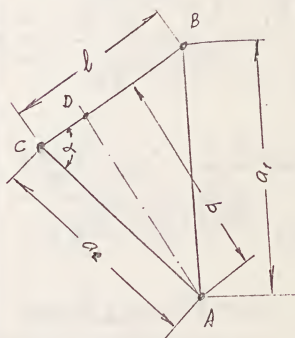


Figura 2

Si unimos los extremos  $C$  y  $B$  (fig. 2) de una arista del poliedro derivado, con el centro  $A$  de su esfera circunscrita, se nos formará el triángulo  $A \cdot B \cdot C$ , de lados  $\underline{CB} = l$ ;  $\underline{AC} = a_2$  y  $\underline{AB} = a_1$ .

La altura  $\underline{AD} = b$ , correspondiente al lado  $\underline{l}$ , será el radio pedido, por lo que

$$b = a_2 \operatorname{sen} \alpha \quad (1)$$

El valor del ángulo  $\alpha$ , se deduce de la fórmula trigonométrica

$$(a_1)^2 = (a_2)^2 + l^2 - 2a_2 l \cos \alpha$$

de la que

$$\cos \alpha = \frac{(a_2)^2 + l^2 - (a_1)^2}{2a_2 l}$$

en la que sustituyendo los valores ya deducidos de

$$a_2 = \sqrt{\frac{3(5-\sqrt{5})}{10}} a_1 \quad \text{y} \quad l = \sqrt{\frac{3-\sqrt{5}}{2}} a_1$$

tendremos que

$$\boxed{\cos \alpha} = \frac{\left(\sqrt{\frac{3(5-\sqrt{5})}{10}} a_1\right)^2 + \left(\sqrt{\frac{3-\sqrt{5}}{2}} a_1\right)^2 - (a_1)^2}{2 \sqrt{\frac{3(5-\sqrt{5})}{10}} a_1 \times \sqrt{\frac{3-\sqrt{5}}{2}} a_1} = \boxed{\sqrt{\frac{5-2\sqrt{5}}{15}} a_1}$$



The following is a list of the  
 names of the persons who  
 have been appointed to the  
 various committees of the  
 Board of Directors of the  
 City of New York, for the  
 year 1911.



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 have been appointed to the  
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 City of New York, for the  
 year 1911.

Desarrollo del cálculo anterior:

$$\begin{aligned}
 \cos \alpha &= \frac{\left(\sqrt{\frac{3(5-\sqrt{5})}{10}} a_1\right)^2 + \left(\sqrt{\frac{3-\sqrt{5}}{2}} a_1\right)^2 - (a_1)^2}{2\sqrt{\frac{3(5-\sqrt{5})}{10}} a_1 \cdot \sqrt{\frac{3-\sqrt{5}}{2}} a_1} = \\
 &= \frac{\frac{3(5-\sqrt{5})}{10} (a_1)^2 + \frac{3-\sqrt{5}}{2} (a_1)^2 - (a_1)^2}{2\sqrt{\frac{3(5-\sqrt{5})(3-\sqrt{5})}{20}} (a_1)^2} = \frac{\frac{15-3\sqrt{5}}{10} + \frac{3-\sqrt{5}}{2} - 1}{2\sqrt{\frac{3(5-\sqrt{5})(3-\sqrt{5})}{20}}} = \\
 &= \frac{\frac{15-3\sqrt{5}+15-5\sqrt{5}-10}{10}}{\sqrt{\frac{3(5-\sqrt{5})(3-\sqrt{5})}{5}}} = \frac{\frac{20-8\sqrt{5}}{10}}{\sqrt{\frac{3(15-3\sqrt{5}-5\sqrt{5}+5)}{5}}} = \frac{\frac{10-4\sqrt{5}}{5}}{\sqrt{\frac{3(20-2\sqrt{5})}{5}}} = \\
 &= \frac{\frac{2(5-2\sqrt{5})}{5}}{\sqrt{\frac{12(5-2\sqrt{5})}{5}}} = \frac{\frac{2}{5}(5-2\sqrt{5})}{2\sqrt{\frac{3}{5}(5-2\sqrt{5})}} = \frac{1}{5} \times \frac{(5-2\sqrt{5})}{\sqrt{\frac{3}{5}(5-2\sqrt{5})}} = \\
 &= \frac{1}{5} \times \sqrt{\frac{(5-2\sqrt{5})^2}{\frac{3}{5}(5-2\sqrt{5})}} = \frac{1}{5} \sqrt{\frac{5(5-2\sqrt{5})}{3}} = \boxed{\sqrt{\frac{5-2\sqrt{5}}{15}}}
 \end{aligned}$$



De aquí se deduce que

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \left(\sqrt{\frac{5-2\sqrt{5}}{15}}\right)^2} = \boxed{\sqrt{\frac{2(5+\sqrt{5})}{15}}}$$

Desarrollo del cálculo anterior:  $\sin \alpha = \sqrt{1 - \left(\sqrt{\frac{5-2\sqrt{5}}{15}}\right)^2} =$ 

$$= \sqrt{1 - \frac{5-2\sqrt{5}}{15}} = \sqrt{\frac{15-5+2\sqrt{5}}{15}} = \sqrt{\frac{10+2\sqrt{5}}{15}} = \boxed{\sqrt{\frac{2(5+\sqrt{5})}{15}}}$$

Valor que sustituido en la expresión inicial (1), nos da

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$$\boxed{b} = a_2 \operatorname{sen} \alpha = \sqrt{\frac{3(5-\sqrt{5})}{10}} a_1 \times \sqrt{\frac{2(5+\sqrt{5})}{15}} = \boxed{\frac{2\sqrt{5}}{5} a_1}$$

Desarrollo del cálculo anterior:

$$\begin{aligned} \boxed{b} &= \sqrt{\frac{3(5-\sqrt{5})}{10}} a_1 \times \sqrt{\frac{2(5+\sqrt{5})}{15}} = \sqrt{\frac{3 \times 2 \times (5-\sqrt{5})(5+\sqrt{5})}{10 \times 15}} a_1 = \sqrt{\frac{5^2-5}{5 \times 5}} a_1 = \\ &= \sqrt{\frac{20}{25}} a_1 = \boxed{\frac{2\sqrt{5}}{5} a_1} \end{aligned}$$

c = Radio de la esfera tangente a las caras

Es igual al radio de la esfera tangente a las aristas de uno de los poliedros dados.

Tomando como base el dodecaedro, su arista, determinada anteriormente en función de  $a_1$ , es

$$l_{12} = 2 \sqrt{\frac{5-2\sqrt{5}}{5}} a_1$$

y el radio de la esfera tangente a las aristas de este dodecaedro, valdrá (ver fórm. 31, lám. 4)

$$\boxed{c} = b_{12} = \frac{3+\sqrt{5}}{4} l_{12} = \frac{3+\sqrt{5}}{4} \times 2 \sqrt{\frac{5-2\sqrt{5}}{5}} a_1 = \boxed{\sqrt{\frac{(5+\sqrt{5})}{10}} a_1}$$

Desarrollo del cálculo anterior:

$$\boxed{c} = \frac{3+\sqrt{5}}{4} \times 2 \sqrt{\frac{5-2\sqrt{5}}{5}} a_1 = \frac{3+\sqrt{5}}{2} \sqrt{\frac{5-2\sqrt{5}}{5}} a_1 = \frac{1}{2} \sqrt{\frac{(5-2\sqrt{5})(3+\sqrt{5})^2}{5}} a_1 =$$

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$$= \frac{1}{2} \sqrt{\frac{(5-2\sqrt{5})(9+5+6\sqrt{5})}{5}} a_1 = \frac{1}{2} \sqrt{\frac{(5-2\sqrt{5})(14+6\sqrt{5})}{5}} a_1 = \frac{1}{2} \sqrt{\frac{2(5-2\sqrt{5})(7+3\sqrt{5})}{5}} a_1 =$$

$$= \frac{1}{2} \sqrt{\frac{2(35-14\sqrt{5}+15\sqrt{5}-30)}{5}} a_1 = \frac{1}{2} \sqrt{\frac{2(5+\sqrt{5})}{5}} a_1 = \boxed{\sqrt{\frac{(5+\sqrt{5})}{10}} a_1}$$

Este mismo valor se obtiene partiendo del icosaedro, cuya arista es

$$l_{20} = \sqrt{\frac{10-2\sqrt{5}}{5}} a_1$$

y el radio de la esfera tangente (ver fórm. 44, lám. 5)

$$\boxed{C} = b_{20} = \frac{1+\sqrt{5}}{4} l_{20} = \frac{1+\sqrt{5}}{4} \times \sqrt{\frac{10-2\sqrt{5}}{5}} a_1 = \boxed{\sqrt{\frac{5+\sqrt{5}}{10}} a_1}$$

Desarrollo del cálculo anterior:  $\boxed{C} = \frac{1+\sqrt{5}}{4} \times \sqrt{\frac{10-2\sqrt{5}}{5}} a_1 =$

$$= \frac{1}{4} \sqrt{\frac{2(5-\sqrt{5})(1+\sqrt{5})^2}{5}} a_1 = \frac{1}{4} \sqrt{\frac{2(5-\sqrt{5})(1+5+2\sqrt{5})}{5}} a_1 =$$

$$= \frac{1}{4} \sqrt{\frac{4(5-\sqrt{5})(3+\sqrt{5})}{5}} a_1 = \frac{1}{2} \sqrt{\frac{15-3\sqrt{5}+5\sqrt{5}-5}{5}} a_1 = \frac{1}{2} \sqrt{\frac{10+2\sqrt{5}}{5}} a_1 =$$

$$= \frac{1}{2} \sqrt{\frac{2(5+\sqrt{5})}{5}} a_1 = \boxed{\sqrt{\frac{5+\sqrt{5}}{10}} a_1}$$

$l_{IV} =$  Distancia entre los centros de dos caras contiguas

En el estudio que hicimos en la lámina 18 del polie-

Example 1: Find the value of  $\sin^{-1}(\sin \frac{\pi}{6})$ .  
 Solution: We know that  $\sin \frac{\pi}{6} = \frac{1}{2}$ .  
 Therefore,  $\sin^{-1}(\sin \frac{\pi}{6}) = \sin^{-1}(\frac{1}{2})$ .  
 We know that  $\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$ .  
 Hence,  $\sin^{-1}(\sin \frac{\pi}{6}) = \frac{\pi}{6}$ .

Example 2: Find the value of  $\cos^{-1}(\cos \frac{2\pi}{3})$ .  
 Solution: We know that  $\cos \frac{2\pi}{3} = -\frac{1}{2}$ .  
 Therefore,  $\cos^{-1}(\cos \frac{2\pi}{3}) = \cos^{-1}(-\frac{1}{2})$ .  
 We know that  $\cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3}$ .  
 Hence,  $\cos^{-1}(\cos \frac{2\pi}{3}) = \frac{2\pi}{3}$ .

Example 3: Find the value of  $\tan^{-1}(\tan \frac{\pi}{4})$ .  
 Solution: We know that  $\tan \frac{\pi}{4} = 1$ .  
 Therefore,  $\tan^{-1}(\tan \frac{\pi}{4}) = \tan^{-1}(1)$ .  
 We know that  $\tan^{-1}(1) = \frac{\pi}{4}$ .  
 Hence,  $\tan^{-1}(\tan \frac{\pi}{4}) = \frac{\pi}{4}$ .

Example 4: Find the value of  $\cot^{-1}(\cot \frac{\pi}{3})$ .  
 Solution: We know that  $\cot \frac{\pi}{3} = \frac{1}{\sqrt{3}}$ .  
 Therefore,  $\cot^{-1}(\cot \frac{\pi}{3}) = \cot^{-1}(\frac{1}{\sqrt{3}})$ .  
 We know that  $\cot^{-1}(\frac{1}{\sqrt{3}}) = \frac{\pi}{3}$ .  
 Hence,  $\cot^{-1}(\cot \frac{\pi}{3}) = \frac{\pi}{3}$ .

Example 5: Find the value of  $\sec^{-1}(\sec \frac{\pi}{2})$ .  
 Solution: We know that  $\sec \frac{\pi}{2}$  is not defined.  
 Therefore,  $\sec^{-1}(\sec \frac{\pi}{2})$  is not defined.

Example 6: Find the value of  $\csc^{-1}(\csc \frac{\pi}{2})$ .  
 Solution: We know that  $\csc \frac{\pi}{2} = 1$ .  
 Therefore,  $\csc^{-1}(\csc \frac{\pi}{2}) = \csc^{-1}(1)$ .  
 We know that  $\csc^{-1}(1) = \frac{\pi}{2}$ .  
 Hence,  $\csc^{-1}(\csc \frac{\pi}{2}) = \frac{\pi}{2}$ .

dro obtenido por la intersección del dodecaedro e icosaedro conjugados por sus aristas, vimos que el sólido común a ambos es un poliedro no regular, convexo, compuesto de 12 pentágonos regulares y 20 triángulos equiláteros de lados de igual longitud.

Este poliedro es un caso particular de los más generales denominados "Poliedros arquimedianos" y lo hemos designado con el nombre de "Arquimediano III".

En la lámina 35 se ha efectuado la representación de dicho Arquimediano III, así como el cálculo analítico de sus principales magnitudes que tienen aplicación a este ejercicio.

En efecto, al unir los centros de dos caras contiguas del poliedro estudiado en esta lámina, se obtiene un "Arquimediano IV" cuyo lado  $l_{IV}$  es la dimensión que deseamos obtener ahora, y cuya esfera circunscrita es coincidente con la esfera tangente a las aristas del dodecaedro e icosaedro dados.

Mas como por otra parte el radio de la esfera circunscrita al Arquimediano IV es igual a (ver lám. 35 fórm. )

$$a_{IV} = \sqrt{\frac{3+\sqrt{5}}{2}} l_{IV}$$

se verificará igualmente (ver lám. 16, fórm. 166), en función del icosaedro:

$$a_{IV} = b'_{20} = \frac{3+\sqrt{5}}{4} l_{12} = \frac{3+\sqrt{5}}{4} \times 2 \sqrt{\frac{5-2\sqrt{5}}{5}} a,$$



por lo que igualando expresiones

$$\sqrt{\frac{3+\sqrt{5}}{2}} l_{IV} = \frac{3+\sqrt{5}}{4} \times 2 \sqrt{\frac{5-2\sqrt{5}}{5}} a_1, \quad \text{de donde}$$

$$\boxed{l_{IV}} = \frac{\frac{3+\sqrt{5}}{4} \times 2 \sqrt{\frac{5-2\sqrt{5}}{5}}}{\sqrt{\frac{3+\sqrt{5}}{2}}} a_1 = \boxed{\sqrt{\frac{5-\sqrt{5}}{10}} a_1}$$

Desarrollo del cálculo anterior:

$$\boxed{l_{IV}} = \frac{\frac{3+\sqrt{5}}{4} \times 2 \sqrt{\frac{5-2\sqrt{5}}{5}}}{\sqrt{\frac{3+\sqrt{5}}{2}}} a_1 = \frac{3+\sqrt{5}}{2} \sqrt{\frac{5-2\sqrt{5}}{5} : \frac{3+\sqrt{5}}{2}} a_1 =$$

$$= \frac{3+\sqrt{5}}{2} \sqrt{\frac{2(5-2\sqrt{5})}{5(3+\sqrt{5})}} a_1 = \frac{1}{2} \sqrt{\frac{2(5-2\sqrt{5})(3+\sqrt{5})^2}{5(3+\sqrt{5})}} a_1 =$$

$$= \frac{1}{2} \sqrt{\frac{2}{5} (5-2\sqrt{5})(3+\sqrt{5})} a_1 = \frac{1}{2} \sqrt{\frac{2}{5} (15-6\sqrt{5}+5\sqrt{5}-10)} a_1 =$$

$$= \sqrt{\frac{1}{10} (5-\sqrt{5})} a_1 = \boxed{\sqrt{\frac{5-\sqrt{5}}{10}} a_1}$$

Al mismo resultado llegaríamos tomando el radio de la esfera tangente a la arista del dodecaedro dado.

Entonces tendríamos (ver lám. 17, fórm. 183)

$$a_{IV} = b'_{12} = \frac{1+\sqrt{5}}{4} l_{20} = \frac{1+\sqrt{5}}{4} \times \sqrt{\frac{10-2\sqrt{5}}{5}} a_1$$



Let  $x = \frac{a}{b}$  and  $y = \frac{c}{d}$  be two rational numbers.

Then,  $x + y = \frac{a}{b} + \frac{c}{d}$

$$= \frac{ad + bc}{bd}$$

Similarly,  $x - y = \frac{a}{b} - \frac{c}{d}$

$$= \frac{ad - bc}{bd}$$

$$xy = \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{x}{y} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

$$\frac{1}{x} = \frac{1}{\frac{a}{b}} = \frac{b}{a}$$

Thus, we have proved that the operations on rational numbers are closed.

Therefore, the set of rational numbers is closed under addition, subtraction, multiplication and division.

que juntamente con la

$$a_{11} = \sqrt{\frac{3+\sqrt{5}}{2}} l_{11}$$

nos permite despejar  $l_{11}$ , así:

$$\boxed{l_{11}} = \frac{\frac{1+\sqrt{5}}{4} \sqrt{\frac{10-2\sqrt{5}}{5}}}{\sqrt{\frac{3+\sqrt{5}}{2}}} a_1 = \boxed{\sqrt{\frac{5-\sqrt{5}}{10}} a_1}$$

Desarrollo del cálculo anterior:

$$\begin{aligned} \boxed{l_{11}} &= \frac{\frac{1+\sqrt{5}}{4} \sqrt{\frac{10-2\sqrt{5}}{5}}}{\sqrt{\frac{3+\sqrt{5}}{2}}} a_1 = \frac{1+\sqrt{5}}{4} \sqrt{\frac{2(5-\sqrt{5})}{5}} \cdot \frac{3+\sqrt{5}}{2} a_1 = \\ &= \frac{1+\sqrt{5}}{4} \sqrt{\frac{4(5-\sqrt{5})}{5(3+\sqrt{5})}} a_1 = \frac{1+\sqrt{5}}{2} \sqrt{\frac{5-\sqrt{5}}{5(3+\sqrt{5})}} a_1 = \frac{1}{2} \sqrt{\frac{(5-\sqrt{5})(1+\sqrt{5})^2}{5(3+\sqrt{5})}} a_1 = \\ &= \frac{1}{2} \sqrt{\frac{(5-\sqrt{5})(1+5+2\sqrt{5})}{5(3+\sqrt{5})}} a_1 = \frac{1}{2} \sqrt{\frac{2(5-\sqrt{5})(3+\sqrt{5})}{5(3+\sqrt{5})}} a_1 = \frac{1}{2} \sqrt{\frac{2}{5}(5-\sqrt{5})} a_1 = \\ &= \boxed{\sqrt{\frac{5-\sqrt{5}}{10}} a_1} \end{aligned}$$

valor coincidente con el anterior

$2\varphi$  = Ángulo rectilíneo del diedro formado por dos caras contiguas.

Si consideramos el diedro formado por dos caras contiguas en una arista cualquiera del poliedro derivado,

Date	Page	No.
<p>             The following is a list of the names of the persons who have been admitted to the office of the Secretary of the Board of Education, during the year ending 1890-1891.         </p> <p>             The names are arranged in alphabetical order, and are given in full, with the date of admission, and the name of the person to whom they were assigned.         </p> <p>             The names are given in full, with the date of admission, and the name of the person to whom they were assigned.         </p>		
1890-1891	1890-1891	1890-1891

y cortamos dicho diedro por un plano perpendicular a la arista, que pase al mismo tiempo por el centro de una de las caras, dicho plano pasará igualmente por el centro de la otra (todas las caras son iguales); las intersecciones de dicho plano con las dos caras, serán los lados del ángulo rectilíneo del diedro.

Si unimos seguidamente los centros de las dos caras, se

nos formará un triángulo isósceles  $\triangle A.B.C$  (fig. 3), cuya base

$\underline{C.B}$  es la magnitud " $l_{IV}$ ",

ya calculada, y los lados iguales la distancia del centro de

la cara a la arista (fig. 2),

o sea la magnitud  $p$  también

obtenida anteriormente. El ángulo  $\underline{A}$  opuesto a la base, será el buscado, por lo que

$$\boxed{\text{sen } \varphi} = \frac{CD}{CA} = \frac{\frac{1}{2} l_{IV}}{p} = \frac{\frac{1}{2} \sqrt{\frac{5-\sqrt{5}}{10}} a_1}{\sqrt{\frac{3-\sqrt{5}}{10}} a_1} = \boxed{\frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{2}}}$$

Desarrollo del cálculo anterior:  $\boxed{\text{sen } \varphi} = \frac{\frac{1}{2} \sqrt{\frac{5-\sqrt{5}}{10}} a_1}{\sqrt{\frac{3-\sqrt{5}}{10}} a_1} =$

$$= \frac{1}{2} \sqrt{\frac{5-\sqrt{5}}{10}} \cdot \frac{3-\sqrt{5}}{10} = \frac{1}{2} \sqrt{\frac{5-\sqrt{5}}{3-\sqrt{5}}} = \frac{1}{2} \sqrt{\frac{(5-\sqrt{5})(3+\sqrt{5})}{4}} =$$

$$= \frac{1}{2} \sqrt{\frac{15-3\sqrt{5}+5\sqrt{5}-5}{4}} = \frac{1}{2} \sqrt{\frac{10+2\sqrt{5}}{4}} = \boxed{\frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{2}}}$$





El valor numérico será

$$\operatorname{sen} \varphi = \frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{2}} = \frac{1}{2} \sqrt{\frac{7.23\ 60\ 67\ 98}{2}} = \frac{1}{2} \sqrt{3.61\ 80\ 33\ 99}$$

$$\lg \operatorname{sen} \varphi = \lg. 0.5 \text{ ————— } 7.698\ 97\ 00$$

$$+ \frac{1}{2} \lg 3.61\ 80\ 33\ 99 = \frac{0.558\ 47\ 27}{2} = + \frac{0.279\ 23\ 63}{1.978\ 20\ 63} = \lg \operatorname{sen} \varphi$$

$$\varphi = 73^\circ$$

$$2\varphi = 144^\circ$$

S = Superficie lateral

El área  $S_1$  de una cara (rombo) se deduce de sus diagonales (fig. 1) y será:

$$\boxed{S_1} = \frac{l_{12} \times l_{20}}{2} = \frac{2\sqrt{\frac{5-2\sqrt{5}}{5}} a_1 \times \sqrt{\frac{10-2\sqrt{5}}{5}} a_1}{2} = \boxed{\frac{3\sqrt{5}-5}{5} (a_1)^2}$$

Desarrollo del cálculo anterior:

$$\begin{aligned} S_1 &= \frac{2\sqrt{\frac{5-2\sqrt{5}}{5}} a_1 \times \sqrt{\frac{10-2\sqrt{5}}{5}} a_1}{2} = \sqrt{\frac{5-2\sqrt{5}}{5} \times \frac{10-2\sqrt{5}}{5}} (a_1)^2 = \\ &= \frac{1}{5} \sqrt{2(5-2\sqrt{5})(5-\sqrt{5})} (a_1)^2 = \frac{1}{5} \sqrt{2(25-10\sqrt{5}-5\sqrt{5}+10)} (a_1)^2 = \\ &= \frac{1}{5} \sqrt{2(35-15\sqrt{5})} (a_1)^2 = \frac{1}{5} \sqrt{2 \times 5(7-3\sqrt{5})} (a_1)^2 = \sqrt{\frac{2(7-3\sqrt{5})}{5}} (a_1)^2 = \\ &= \sqrt{\frac{2}{5}} \times \sqrt{7-3\sqrt{5}} (a_1)^2 = \quad \text{y por ser } 4^2 - (3\sqrt{5})^2 = 4, \text{ tendremos} \end{aligned}$$

Date	Particulars	Amount
	To Balance b/d	100.00
	By Cash	50.00
	By Bank	20.00
	By Sales	30.00
	By Interest	10.00
	By Dividend	5.00
	By Profit	15.00
	By Other	5.00
	By Total	135.00
	By Balance c/d	135.00
	By Total	135.00
	By Total	135.00
	By Total	135.00
	By Total	135.00
	By Total	135.00

$$= \frac{\sqrt{2}}{5} \times \left( \sqrt{\frac{9}{2}} - \sqrt{\frac{5}{2}} \right) (a_1)^2 = \left( \sqrt{\frac{18}{10}} - \sqrt{\frac{10}{10}} \right) (a_1)^2 = \left( \sqrt{\frac{9}{5}} - 1 \right) (a_1)^2 =$$

$$= \left( \frac{3}{\sqrt{5}} - 1 \right) (a_1)^2 = \left( \frac{3\sqrt{5}}{5} - 1 \right) (a_1)^2 = \frac{3\sqrt{5} - 5}{5} (a_1)^2$$

y la superficie total del poliedro (30 caras)

$$\boxed{S} = 30 \times \frac{3\sqrt{5} - 5}{5} (a_1)^2 = \boxed{6 (3\sqrt{5} - 5) (a_1)^2}$$

V = Volumen

Si consideramos unidos los vértices de una cara, con el centro de la esfera circunscrita al poliedro derivado, se nos formará una pirámide recta de base cuadrada y altura igual al radio  $c$  de la esfera inscrita. El volumen  $V_1$  de dicha pirámide será pues

$$\boxed{V_1} = S_1 \times \frac{1}{3} c = \left( \frac{3\sqrt{5} - 5}{5} (a_1)^2 \right) \times \frac{1}{3} \sqrt{\frac{5 + \sqrt{5}}{10}} (a_1) = \boxed{\frac{2}{15} \sqrt{5 - 2\sqrt{5}} (a_1)^3}$$

Desarrollo del cálculo anterior:  $\boxed{V_1} = \left[ \frac{3\sqrt{5} - 5}{5} (a_1)^2 \right] \times \frac{1}{3} \sqrt{\frac{5 + \sqrt{5}}{10}} (a_1) =$

$$= \frac{3\sqrt{5} - 5}{5} \times \frac{1}{3} \times \sqrt{\frac{5 + \sqrt{5}}{10}} (a_1)^3 = \frac{3\sqrt{5} - 5}{15} \sqrt{\frac{(5 + \sqrt{5})}{10}} (a_1)^3 = \frac{1}{15} \sqrt{\frac{(5 + \sqrt{5})(3\sqrt{5} - 5)^2}{10}} (a_1)^3 =$$

$$= \frac{1}{15} \sqrt{\frac{(5 + \sqrt{5})(45 + 25 - 30\sqrt{5})}{10}} (a_1)^3 = \frac{1}{15} \sqrt{\frac{(5 + \sqrt{5})(70 - 30\sqrt{5})}{10}} (a_1)^3 =$$

$$= \frac{1}{15} \sqrt{(5 + \sqrt{5})(7 - 3\sqrt{5})} (a_1)^3 = \frac{1}{15} \sqrt{35 + 7\sqrt{5} - 15\sqrt{5} - 15} (a_1)^3 =$$

Let  $f(x) = \frac{1}{x^2}$  and  $g(x) = \frac{1}{x^3}$  be two functions.

$$f'(x) = \frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$$

$$g'(x) = \frac{d}{dx} x^{-3} = -3x^{-4} = -\frac{3}{x^4}$$

Now we will find the derivative of  $f(x)g(x)$ .

$$\frac{d}{dx} \left( \frac{1}{x^2} \cdot \frac{1}{x^3} \right) = \frac{1}{x^5} \cdot \left( -\frac{2}{x^3} - \frac{3}{x^4} \right) = -\frac{2}{x^8} - \frac{3}{x^9}$$

Ans:  $-\frac{2}{x^8} - \frac{3}{x^9}$

The above result can be verified by using the product rule. Let  $u = \frac{1}{x^2}$  and  $v = \frac{1}{x^3}$ . Then  $u' = -\frac{2}{x^3}$  and  $v' = -\frac{3}{x^4}$ . According to the product rule,  $(uv)' = u'v + uv'$ . So,  $\left(\frac{1}{x^2} \cdot \frac{1}{x^3}\right)' = \left(-\frac{2}{x^3}\right) \cdot \frac{1}{x^3} + \frac{1}{x^2} \cdot \left(-\frac{3}{x^4}\right) = -\frac{2}{x^6} - \frac{3}{x^6} = -\frac{5}{x^6}$ . Wait, this seems to be different from the previous result. Let me re-calculate.  $\frac{1}{x^2} \cdot \frac{1}{x^3} = \frac{1}{x^5}$ . So,  $\frac{d}{dx} \left( \frac{1}{x^5} \right) = -5x^{-6} = -\frac{5}{x^6}$ . Yes, the previous result was incorrect. The correct result is  $-\frac{5}{x^6}$ .

$$\frac{d}{dx} \left( \frac{1}{x^5} \right) = -5x^{-6} = -\frac{5}{x^6} \quad \square$$

Let  $f(x) = \frac{1}{x^2}$  and  $g(x) = \frac{1}{x^3}$  be two functions.

$$f'(x) = \frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$$

$$g'(x) = \frac{d}{dx} x^{-3} = -3x^{-4} = -\frac{3}{x^4}$$

Now we will find the derivative of  $f(x)g(x)$ .

$$\frac{d}{dx} \left( \frac{1}{x^2} \cdot \frac{1}{x^3} \right) = \frac{1}{x^5} \cdot \left( -\frac{2}{x^3} - \frac{3}{x^4} \right) = -\frac{2}{x^8} - \frac{3}{x^9}$$

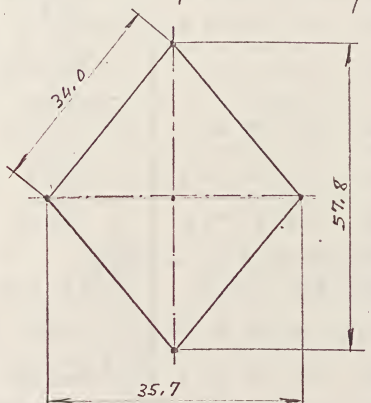
$$= \frac{1}{15} \sqrt{20-2\sqrt{5}} (a_1)^3 = \frac{2}{15} \sqrt{5-2\sqrt{5}} (a_1)^3$$

Y el volumen  $V$  total del poliedro (30 pirámides iguales)

$$V = 30 \times \frac{2}{15} \sqrt{5-2\sqrt{5}} (a_1)^3 = 4 \sqrt{5-2\sqrt{5}} (a_1)^3$$

### FIGURA CORPÓREA

Se obtiene por acoplamiento de 30 rombos, cuyas diagonales son los lados de los poliedros conjugados dados.



La longitud de la arista (34,0 mm) sirve de comprobación al trazado.



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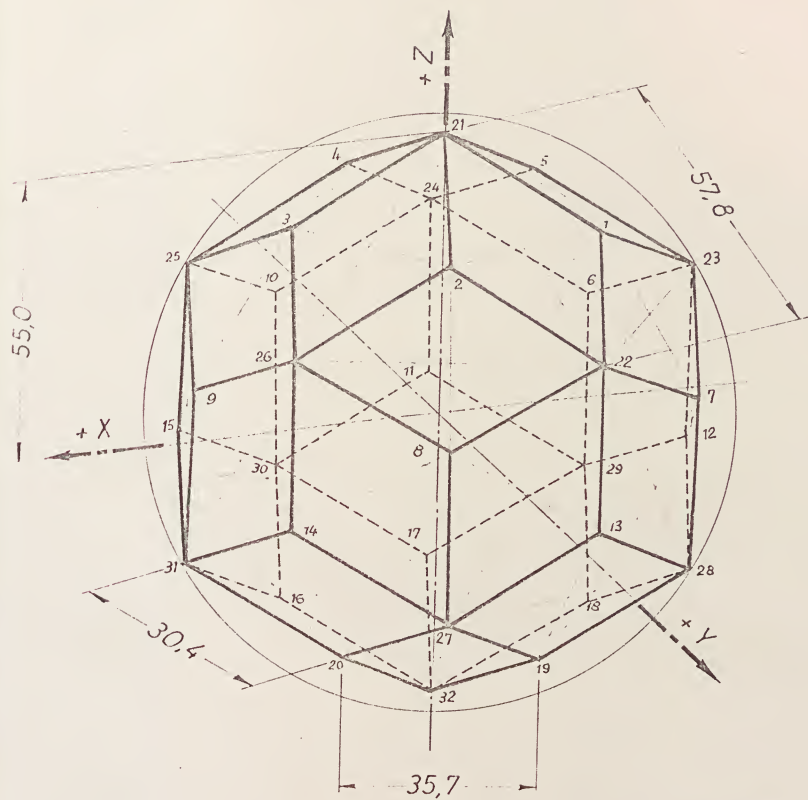


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En el cuadro sinóptico que damos a continuación, resumimos los resultados anteriores:

Magnitud	Valor exacto	Valor decimal aproximado
$l$	$\sqrt{\frac{3-\sqrt{5}}{2}} a_1$	0,61 80 34... $a_1$
$d_2$	$\sqrt{\frac{3(5-\sqrt{5})}{10}} a_1$	0,91 05 93... $a_1$
$b$	$\frac{2\sqrt{5}}{5} a_1$	0,89 44 27... $a_1$
$c$	$\sqrt{\frac{5+\sqrt{5}}{10}} a_1$	0,85 06 51... $a_1$
$l_{20}$	$\sqrt{\frac{10-2\sqrt{5}}{5}} a_1$	1,05 14 62... $a_1$
$l_{12}$	$2\sqrt{\frac{5-2\sqrt{5}}{5}} a_1$	0,64 98 39... $a_1$
$l_{14}$	$\sqrt{\frac{5-\sqrt{5}}{10}} a_1$	0,52 57 31... $a_1$
$P$	$\sqrt{\frac{3-\sqrt{5}}{10}} a_1$	0,27 63 93... $a_1$
$2\varphi$	$\text{sen } \varphi = \frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{2}}$	$\log. \text{ sen } \varphi = 7,978\ 20\ 63$ $\varphi = 72^\circ$ $2\varphi = 144^\circ$
$S$	$6(3\sqrt{5}-5) (a_1)^2$	10,24 72 24... $(a_1)^2$
$V$	$4\sqrt{5-2\sqrt{5}} (a_1)^3$	2,90 61 70... $(a_1)^3$





Derivado de los conjugados dodecaedro-icosaedro



Original manuscript of the author of the book









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